ABSTRACT

A Perturbation Theory for Hamilton's Principal Function: Applications to Boundary Value Problems

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This thesis introduces an analytical perturbation theory for Hamilton's principal function and Hamilton's characteristic function. Based on Hamilton's principle and the research carried out by Sir William Rowan Hamilton, a perturbation theory is developed to analytically solve two-point boundary value problems. The principal function is shown to solve the two-point boundary value problem through simple differentiation and elimination. The characteristic function is related to the principal function through a Legendre transformation, and can also be used to solve two-point boundary value problems. In order to obtain the solution to the perturbed two-point boundary value problem the knowledge of the nominal solution is sufficient. The perturbation theory is applied to the two body problem to study the perturbed dynamics in the vicinity of the Hohmann transfer. It is found that the perturbation can actually offer a lower cost two-impulse transfer to the target orbit than the Hohmann transfer. The numerical error analysis of the perturbation theory is shown for different orders of calculation.

Coupling Hamilton's principal and characteristic functions yields an analytical perturbation theory for the initial value problem, where the state of the perturbed system can be accurately obtained. The perturbation theory is applied to the restricted three-body problem, where the system is viewed as a two-body problem perturbed by the presence of a third body. It is shown that the first order theory can be sufficient to solve the problem, which is expressed in terms of Delaunay elements. The solution to the initial value problem is applied to derive a Keplerian periapsis map that can be used for low-energy space mission design problems.