ABSTRACT

THE HAMILTON-JACOBI THEORY FOR SOLVING TWO-POINT BOUNDARY VALUE PROBLEMS: THEORY AND NUMERICS WITH APPLICATION TO SPACECRAFT FORMATION FLIGHT, OPTIMAL CONTROL AND THE STUDY OF PHASE SPACE STRUCTURE

by

Vincent M. Guibout

Co-Chairs: Daniel J. Scheeres and Anthony M. Bloch

This dissertation has been motivated by the need for new methods to address complex problems that arise in spacecraft formation design. As a direct result of this motivation, a general methodology for solving two-point boundary value problems for Hamiltonian systems has been found. Using the Hamilton-Jacobi theory in conjunction with the canonical transformation induced by the phase flow, it is shown that generating functions solve two-point boundary value problems. Traditional techniques for addressing these problems are iterative and require an initial guess. The method presented in this dissertation solves boundary value problems at the cost of a single function evaluation, although it requires knowledge of at least one generating function. Properties of this method are presented. Specifically, we show that it includes perturbation theory and generalizes it to nonlinear

systems. Most importantly, it predicts the existence of multiple solutions and allows one to recover all of these solutions.

To demonstrate the efficiency of this approach, an algorithm for computing the generating functions is proposed and its convergence properties are studied. As the method developed in this work is based on the Hamiltonian structure of the problem, particular attention must be paid to the numerics of the algorithm. To address this, a general framework for studying the discretization of certain dynamical systems is developed. This framework generalizes earlier work on discretization of Lagrangian and Hamiltonian systems on tangent and cotangent bundles respectively. In addition, it provides new insights into some symplectic integrators and leads to a new discrete Hamilton-Jacobi theory. Most importantly, it allows one to discretize optimal control problems. In particular, a discrete maximum principle is presented.

This dissertation also investigates applications of the proposed method to solve two-point boundary value problems. In particular, new techniques for designing spacecraft formation flight, reconfiguring a formation, and searching for stable configurations in a general dynamical environment are presented. In addition, the present work allows one to reduce the search for periodic orbits with specified periods or locations to solving a set of nonlinear equations. Finally, a novel approach for solving optimal control problems is derived and applied.