ABSTRACT

THE HAMILTON-JACOBI THEORY FOR SOLVING OPTIMAL FEEDBACK CONTROL PROBLEMS WITH GENERAL BOUNDARY CONDITIONS

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This dissertation presents a general methodology for solving the optimal feedback control problem in the context of Hamiltonian system theory. It is first formulated as a two point boundary value problem for a standard Hamiltonian system, and the associated phase flow is viewed as a canonical transformation. Then relying on the Hamilton-Jacobi theory, we employ generating functions to develop a unified methodology for solving a variety of optimal feedback control formulations with general types of boundary conditions.

The major accomplishment is to establish a theoretical connection between the optimal cost function and a special kind of generating function. Guided by this recognition, we are ultimately led to a new flexible representation of the optimal feedback control law for a given system, which is adjustable to various types of boundary conditions by algebraic conversions and partial differentiations. This adaptive property provides a substantial advantage over the classical dynamic programming method in the sense that we do not need to solve the Hamilton-Jacobi-Bellman equation repetitively for varying types of boundary conditions. Furthermore for a special type

of boundary condition, it also enables us to work around an inherent singularity of the Hamilton-Jacobi-Bellman equation by a special algebraic transformation.

Taking full advantage of these theoretical insights, we develop a systematic algorithm for solving a class of optimal feedback control problems represented by smooth analytic Hamiltonians, and apply it to problems with different characteristics. Then, broadening the practical utility of generating functions for problems where the relevant Hamiltonian is non-smooth, we construct a pair of Cauchy problems from the associated Hamilton-Jacobi equations. This alternative formulation is justified by solving problems with control constraints which usually feature non-smoothness in the control logic.

The main result of this research establishes that the optimal feedback control problem can be solved by the generating functions of the canonical solution flow corresponding to the necessary conditions. This result demonstrates the power of analyzing the optimal feedback control problem within the comprehensive field of classical Hamiltonian system theory.