Station-keeping Analysis for the Terrestrial Planet Formation in Halo Orbit

Kathryn E. Hamera,* Jeffrey S. Parker,† and George H. Born‡

Colorado Center for Astrodynamics Research, Boulder, CO, 80309, USA

Martin W. Lo§

Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109

The proposed Terrestrial Planet Finder Interferometer (TPF-I) mission with the L2 option will implement Station-keeping Maneuvers (SKMs). These SKMs will correct the position and velocity errors that the satellites accumulate while performing numerous daily formation maneuvers on an unstable halo orbit. A series of simulations was conducted to determine the frequency and magnitude of the SKMs. Simulations were conducted using specified error parameters. The worst set of errors was drawn from Gaussian distributions with standard deviations of 10 km for position orbit determination errors and 1 cm/s for velocity orbit determination errors. Every formation control, reconfiguration, and station-keeping maneuver also included the addition of errors where the largest errors were drawn from Gaussian distributions with standard deviations of 5%, 5%, and 2% of the maneuver components in the x-, y-, and z-directions, respectively. The average of ten simulations showed the SKM budget was approximately 70 cm/s per satellite per year. Of these simulations, the highest budget was found to be 110 cm/s per satellite per year. In all test cases, SKMs were executed every 60 days. These results show that the station-keeping requirements of the L2 option for the TPF-I mission yield a very reasonable ∆V budget.

Nomenclature

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>CRTBP</td>
<td>Circular Restricted Three Body Problem</td>
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<tr>
<td>EL2</td>
<td>2nd Sun-Earth Libration Point</td>
</tr>
<tr>
<td>OD</td>
<td>Orbit Determination</td>
</tr>
<tr>
<td>SKM</td>
<td>Station-keeping Maneuver</td>
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<tr>
<td>t₀</td>
<td>A given initial time</td>
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<tr>
<td>tₙ</td>
<td>A given final time</td>
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<tr>
<td>tᵢ</td>
<td>The time at some intermediate state i</td>
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<tr>
<td>TPF-C</td>
<td>Terrestrial Planet Finder - Coronagraph</td>
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<td>TPF-F</td>
<td>Terrestrial Planet Finder - Interferometer</td>
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I. Introduction

Terrestrial Planet Finder is a combination of two proposed space missions: TPF-C, a visible-light coronagraph and TPF-I, a formation flying mid-infrared interferometer. TPF-C will be equipped with onboard coronagraph instrumentation that will have the capability to block the glare of a star to allow the

*Doctoral Student, The Colorado Center for Astrodynamics Research, University of Colorado, Boulder, CO 80309, and Student AIAA Member.
†Doctoral Student, The Colorado Center for Astrodynamics Research, University of Colorado, Boulder, CO 80309, and Student AIAA Member.
‡Director, The Colorado Center for Astrodynamics Research, University of Colorado, Boulder, CO 80309, and Fellow AIAA and AAS.
§Member Technical Staff, Navigation and Mission Design, AIAA Member

American Institute of Aeronautics and Astronautics
detection and characterization of dim planets that may be in orbit about the star.\textsuperscript{1} The TPF-I mission will use interferometry to combine the infrared light detected by multiple telescopes to simulate a much larger telescope. This will enable the detection and study of individual planets orbiting stars observed by the TPF-C mission. TPF-I will also be able to observe stars and planets beyond the reach of TPF-C. The two missions will provide definitive characterization of extra-solar planets and planetary systems and yield a reliable and robust assessment of habitability and presences of the signs of life.\textsuperscript{2}

There are two proposed orbit options for the TPF-I mission: An Earth drift-away orbit, and a halo orbit about the second Sun-Earth libration point, EL\textsubscript{2}. This paper focuses on the station-keeping requirements necessary to maintain formation flight of the satellite constellation for the EL\textsubscript{2} orbit option. The proposed TPF-I formation consists of five spacecraft located geometrically in a plane: four collectors evenly spaced in a line and one combiner offset perpendicularly from that line, as shown in Figure 1. The distance, \( D \), between the two outer spacecraft was set to 60 meters for the simulations, so that a helix with a 30 meter radius was created. The plane is oriented such that its normal is facing a target star. The center of the line of collectors is located on the nominal orbit, which is a halo orbit\textsuperscript{3} about EL\textsubscript{2} with a period of approximately 178 days. For this study, the TPF-I formation rotates in the plane once every 24 hours following the path of a 100-gon, creating a 100-sided helix about the nominal orbit. Every maneuver that the satellites perform to follow this helical trajectory includes some amount of error. As the errors accumulate, the formation drifts away from its nominal orbit, eventually drifting away at an exponential rate. The purpose of this study is to determine the total cost of the station-keeping maneuvers (SKMs) that will be required to compensate for the accumulation of errors throughout the formation’s mission. Software was created to simulate satellites on a helical trajectory in order to determine the \( \Delta V \) budget required of the TPF-I mission. The assumptions used in the simulations will be outlined in Section I.B.

I.A. The Circular Restricted Three Body Problem

The circular restricted three-body problem (CRTBP) has been used to model the orbits in this study. The three bodies of interest in the system are the Sun, the Earth-Moon Barycenter, and the spacecraft. For simplicity, we will refer to the Earth-Moon Barycenter as the Earth. The mass of the spacecraft is assumed to be negligible, and therefore Keplerian laws may be applied to the motion of the Sun and the Earth. These two massive bodies are assumed to rotate in circular orbits about the center of mass of the system, known as the barycenter.\textsuperscript{4,5} The \( x \)-axis, also known as the syzygy axis,\textsuperscript{6} extends from the origin through the Earth, the \( z \)-axis extends in the direction of the angular momentum of the system, and the \( y \)-axis completes the right-hand coordinate frame. The system is normalized such that the sum of the masses, the distance between the primaries, and the gravitation parameter all equal unity and the orbital period is normalized to \( 2\pi \). These values are normalized by a three-body parameter \( \mu \), defined as the ratio of the smaller primary’s mass (\( M_2 \)) to the sum of the mass of the two primaries (\( M_1 + M_2 \)).\textsuperscript{6} Once the system has been normalized, the coordinates of the Sun and Earth in the rotating axes are therefore equal to \([\text{−}\mu, 0]\) and \([1 − \mu, 0] \), respectively. With the normalized system, the equations of motion for the third body in the rotating frame are given by:

\[
\begin{align*}
\ddot{x} - 2\dot{y} &= x - (1-\mu)\frac{x-x_1}{r_1^3} - \frac{\mu}{r_2^3} \frac{x-x_2}{r_2^3} \\
\dot{y} + 2\dot{x} &= \left(1 - \frac{1-\mu}{r_1^3} - \frac{\mu}{r_2^3}\right)y \\
\ddot{z} &= -\left(\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3}\right)z
\end{align*}
\]

where \( r_1 \) and \( r_2 \) are equal to the distance from the third body to the Sun and Earth, respectively:

\[
\begin{align*}
\frac{r_1^2}{r_2^2} &= (x + \mu)^2 + y^2 \\
\frac{r_2^2}{r_2^2} &= (x - 1 + \mu)^2 + y^2
\end{align*}
\]

Several non-dimensional units have been used in the formation of this model. The system coordinates have been normalized above by the three-body parameter \( \mu \). In this study, the value of \( \mu \) is approximately equal to \( 3.0404 \times 10^{-6} \). Furthermore, distance and time units have been normalized. One distance unit is
the distance between the primary bodies, approximately equal to 1.496 × 10^8 km, and 2π time units is equal to the orbital period of the primaries about the barycenter, approximately 365.24 days.

I.B. Simulation Assumptions

Several assumptions have been used in the TPF-I simulation software. It has been assumed that an adequate analysis for a preliminary station-keeping budget could be performed under the assumptions of the CRTBP. The model may be made more realistic later by the implementation of the full JPL ephemeris model of the solar system. The only forces assumed to be acting on the formation are those attributed to the primary and secondary bodies in the CRTBP. Out-gassing, solar radiation pressure, or other perturbations may be added to the model at a later time.

The proposed TPF formation rotates in a plane facing a target star once every 24 hours following a helical trajectory. It has been assumed that the helix may be approximated as an n-gon where n has been set to a value of 100 for this study. The center of the formation lies along the nominal halo orbit and the formation is assumed to rotate about this center with the axis of rotation being normal to the plane formed by the formation. The two outer spacecraft will trace out slightly different helical trajectories than the two inner spacecraft or the combiner. However, it has been assumed that the differences in SKMs will be negligible due to the small relative magnitude in the distances of the spacecraft to the nominal halo orbit. Therefore, in each of the simulations, only the combiner spacecraft has been simulated and it will approximate the overall budget for any of the spacecraft in the formations. Furthermore, it has been assumed that the target star is always located in the positive x direction in the CRTBP frame. In the final simulations, these assumptions will be neglected and the formation will be allowed to rotate to view other stars not in the positive x direction.

A differential corrector has been used to target the position on the nominal halo orbit and the vertices of the 100-gon for computing the correction trajectories and subsequent SKMs. Errors have been added to each formation control and station-keeping maneuver. In addition, errors have been added into the simulation due to orbit determination errors. These maneuvers errors have been assumed to be Gaussian with zero mean and some specified standard deviation. The errors are further discussed in Section II.E.

II. Station-keeping Algorithm

The following section outlines the procedure used to create the sequence of maneuvers necessary for the formation to trace out the helix and the subsequent method used to correct the drift of the constellation due to maneuver execution and orbit determination error accumulation.

II.A. The Nominal Halo Orbit

The proposed TPF-I mission will perform approximately 100 maneuvers per day to create a helical trajectory relative to the nominal halo orbit. To simulate the mission, the nominal halo orbit is first constructed and then the helical trajectory is placed over that nominal orbit. First, points on the nominal halo orbit have been generated for each time step \( t_i \), in the interval from \( t_0 \) to \( t_1 \). The halo orbit is constructed using the method outlined by Howell using the initial conditions shown in Table 1. The period of this halo orbit is approximately 178 days.

<table>
<thead>
<tr>
<th>Table 1. Initial conditions for the halo orbit at time ( t_0 ) in non-dimensional, normalized units.</th>
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<tbody>
<tr>
<td><strong>Position Coordinates</strong></td>
</tr>
<tr>
<td>( x_0 )</td>
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<tr>
<td>( y_0 )</td>
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<tr>
<td>( z_0 )</td>
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</table>
II.B. Helix Creation

The five spacecraft rotate about their nominal halo orbit in a helix, such that the normal vector of the formation’s plane is pointing at a target star. The spacecraft are not actually moving in a helix, but rather an 100-gon that approximates a helix.

A sequence of maneuvers is constructed for each spacecraft, where each maneuver is constructed to transfer the spacecraft from one vertex of the 100-gon to the next at the appropriate time. Each maneuver is determined by targeting the successive vertex of the 100-gon using a differential corrector, as described below. The differential corrector propagates the spacecraft along a real trajectory in the system, rather than assuming that the spacecraft travels in a straight line from vertex to vertex; hence the simulation is more accurate than simply assuming that the spacecraft can move along straight segments of a 100-gon.

The vertices of the 100-gon are determined in the following manner. First, a local coordinate system is set up such that \(x'\) is a unit vector in the direction of the combiner from the nominal orbit, \(y'\) is a unit vector along the line of the collectors, \(z'\) is a unit vector in the direction of the target star, and the coordinate frame follows the satellites along their nominal orbit; this rotating relative frame is shown in Figure 1.

![Figure 1. TPF-I formation satellite positions in the local coordinate x', y', z' frame.](image)

The \(x', y'\) coordinates of the satellites in the local rotating frame are then given as follows:

\[
\begin{align*}
\text{Collector } \#1 &= \left[0, -\frac{D}{2}\right] \\
\text{Collector } \#2 &= \left[0, -\frac{D}{6}\right] \\
\text{Collector } \#3 &= \left[0, \frac{D}{6}\right] \\
\text{Collector } \#4 &= \left[0, \frac{D}{2}\right] \\
\text{Combiner} &= \left[\frac{D}{6 \tan 25^\circ}, 0\right] = \left[\frac{D}{2.80}, 0\right]
\end{align*}
\]

In the local rotating coordinate frame, the position of each satellite does not change from vertex to vertex along the helix. Three transformations are performed to move the coordinates of each satellite from this local rotating coordinate frame into the CRTBP synodic frame. First, the satellite coordinates are transferred into a frame that does not rotate with the helix, but still follows the nominal halo orbit, i.e., the local non-rotating coordinate frame. This frame is still oriented such that the \(z\)-axis is pointed at the target star. Second, the local non-rotating coordinate frame is rotated such that its axes line up with the CRTBP axes. Third, the coordinates are translated into the standard synodic reference frame. These transformations will now be discussed in detail.

The first transformation from the local rotating coordinate frame into the CRTBP is to move into a non-rotating relative coordinate frame. This transformation simply involves rotating the formation about the \(z\)-axis by an angle \(\theta_i\), where \(\theta_i\) defines the angle of the \(i^{th}\) vertex of the 100-gon:
\[ \theta_i = \omega t_i \]  

The angular velocity of the formation about the nominal halo orbit, \( \omega \), is given by:

\[ \omega = \frac{2\pi}{P} \]  

where \( P \) is the period of rotation of the formation about the helix, equal to 24 hours in this study. The coordinates of the satellites at the \( i^{th} \) vertex of the helix in the local non-rotating reference frame are then equal to \( x'_i', y'_i', \) and \( z'_i' \):

\[
\begin{align*}
    x'_i' &= r_x \cos \theta_i + r_y \sin \theta_i \\
    y'_i' &= -r_x \sin \theta_i + r_y \cos \theta_i \\
    z'_i' &= 0
\end{align*}
\]  

where \( r_x \) and \( r_y \) are the \( x' \) and \( y' \) coordinates of the satellites given in Eqs. 4 and 5.

The second transformation is to rotate the coordinates in the local non-rotating reference frame given in Eqs. 8 into a reference frame whose axes are coincident with the CRTBP axes. In this simulation, the formation is targeting a star that is located in the positive \( x \)-direction in the CRTBP frame. Hence, the \( z'' \)-axis is coincident with the CRTBP’s \( x \)-axis. The \( x'' \) and \( y'' \)-axes may be oriented in any configuration based on the orientation of the formation at time \( t_0 \). For this study, at \( t_0 \) the reference frames are oriented with respect to each other as shown in Figure 2. It is important to note that the \( x, z' \) and \( z'' \) axes do not line up in general; this only occurs when observing stars in the \( x \) direction.

It is apparent that the transformation may be completed using two rotations: first, a rotation of \(-90\) degrees about the \( x'' \)-axis and then a rotation of \(-90\) degrees about the \( y'' \)-axis. The result is that the coordinates of the satellite, \( dx, dy, \) and \( dz \), are in a frame that is coincident with the CRTBP frame, but which follows the formation along the nominal orbit. The coordinates \( dx_i, dy_i, \) and \( dz_i \) are found as follows:

\[
\begin{align*}
    dx_i &= 0 \\
    dy_i &= r_x \cos \theta_i + r_y \sin \theta_i \\
    dz_i &= -r_x \sin \theta_i + r_y \cos \theta_i
\end{align*}
\]  

Finally, the third transformation is to translate the coordinates given in Eqs 9 into the CRTBP axes. This is accomplished by adding the states of the nominal halo orbit to the coordinates as follows:

\[
\begin{align*}
    x_{\text{vertex},i} &= x_{\text{halo},i} + dx_i \\
    y_{\text{vertex},i} &= y_{\text{halo},i} + dy_i \\
    z_{\text{vertex},i} &= z_{\text{halo},i} + dz_i
\end{align*}
\]
Thus, given all points on the halo between some $t_0$ and $t_1$, all helix vertices may be calculated. The Howell-Pernicka-Wilson two-level differential corrector\(^7\) has been used to calculate the maneuver necessary to reach the $i^{th}$ state (helix vertex) at the time $t_i$ starting from the state at $t_{i-1}$. The maneuver sequence from $t_0$ to $t_1$ may then be stored and used in the simulation. Figure 3 shows a segment of the nominal helix. The radius of the helix has been increased to 300,000 km from the nominal TPF-I study value of 30 m in order to better graphically illustrate the helix structure. In the following figures, helical trajectories will be indicated by solid lines. The center of the helical trajectories will be indicated dashed lines of the same color. The colors will organized as follows: red will represent the nominal trajectories, black will indicate the trajectories with errors, and blue will represent the correction trajectories.

II.C. Helix Drift

If all of the formation maneuvers were executed perfectly, the center of the helix shown in Figure 3 would be perfectly aligned with the nominal halo orbit indicated by the dashed red line. However, the maneuvers will have some execution error, as well as associated orbit determination errors. Random errors selected from Gaussian distributions with a specified standard deviation have been added to each helix maneuver in the sequence. The addition of random errors is further discussed in Section II.E. The result of this error inclusion is that the helix drifts from the nominal helix trajectory calculated previously. The drifted helix is shown in Figure 4. This black helical trajectory is the result of the accumulation of random maneuver and orbit determination errors. The dashed black line represents the center of the drifted helix, which was calculated by reversing the process used to create the helix maneuvers, discussed in Section II.B. The helix center no longer follows the nominal halo orbit, and an SKM is now necessary to correct the drifted helix back to the nominal halo orbit.

II.D. SKM Calculations

In order to calculate an SKM, it is first necessary to create a correction trajectory from the center of the drifted helix back to the nominal halo orbit. This process is illustrated in Figure 5. The center of the drifted helix is calculated from times $t_0$ to $t_1$ using the drifted helix state and Eqs 10; it is represented by the black dashed line in Figure 5. Points on the nominal halo orbit have been generated for the time interval $t_1$ to $t_2$. The differential corrector is used calculate the maneuver necessary to correct the drift from the center of the drifted helix at time $t_1$ by targeting the state of the nominal halo orbit at time $t_2$, known as the SKM Target. Once the maneuver has been calculated, a trajectory is constructed from the center of the drifted helix at time $t_1$ to the nominal halo orbit at time $t_2$. A correction helix is built around the center of this correction trajectory using Eqs 10. Errors will be added to this new helix sequence, a new drifted center will be calculated, and the entire process will be repeated until the final simulation time is reached. In order to minimize $\Delta V$ costs, the SKM itself is combined vectorally with the first helix maneuver in the sequence. Hence, the SKM takes the spacecraft from a vertex on the drifted orbit to a new vertex that follows a correction trajectory. The black circle in Figure 5 is the point where the SKM is implemented at
Figure 4. The black helical trajectory has drifted from the nominal helix (red) due to the accumulation of maneuver execution and orbit determination errors. The center of the drifted helix (dashed black) is no longer aligned with the nominal halo orbit (dashed red).

It can be seen that this is where the drifted helix seamlessly joins with the correction helix. This minimizes the $\Delta V$ costs while maintaining the desired helical structure for the formation.

Figure 5. The drifted helical trajectory (black) is seamlessly joined to the correction helix (blue) to correct the formation back to the nominal halo orbit (red). The figure on the right shows the full trajectory for one station keeping period, and the figure on the left zooms to the SKM execution point to illustrate the seamless trajectory.

II.E. Maneuver and Orbit Determination Errors

There are two types of errors added into the TPF-I simulations; maneuver execution errors and orbit determination errors. These two types of errors will be described in this section.

II.E.1. Maneuver Errors

In the TPF-I simulations, maneuver errors are added to every formation and station-keeping maneuver to account for imperfect execution capabilities. The maneuver errors are drawn from Gaussian distributions with zero mean and a standard deviation based on a percentage of the components of the maneuvers in the $x$, $y$, and $z$-directions. The percentages of the maneuvers are varied in the study to determine the budget based on the accuracy of the maneuvers.

II.E.2. Orbit Determination Errors

Orbit Determination (OD) errors are the result of imperfect tracking knowledge of the satellites and imperfect mathematical models. The true state of the center of the drifted helix is unknown; hence the helix vertices
and helix maneuver sequences are calculated based on the best estimate of the state propagated forward in a correction trajectory. To account for this imperfect knowledge of the state, OD errors are added to the state of the first helix vertex before the trajectory is propagated forward with the maneuver sequence. Therefore, the satellites will execute a maneuver sequence based upon a correction trajectory that does not start from the actual state of the formation. Multiple simulations are run which vary the position and velocity OD errors. The position OD errors are drawn from Gaussian distributions with zero mean and standard deviations ranging from 10 km to 1 m. Velocity OD errors are drawn from Gaussian distributions with zero mean and standard deviations ranging from 1 cm/s to 0.01 cm/s. These OD errors are based on previous missions such as the Microwave Anisotropy Probe (MAP) mission. MAP was launched in 2001, inserted into a small Lissajous orbit about the EL$_2$ and had RSS position OD errors of 6.7 km and RSS velocity OD errors of nearly 4 mm/s.

III. Algorithm Verification

Simulations have been conducted to verify the capability of the software to correctly construct trajectories from the drifted helix to the nominal halo orbit. In the verification simulations, 10 formation control maneuvers were performed each day to create a helix with a diameter of 30 m about the nominal halo orbit. SKMs were executed perfectly every 15 days over the course of the 178 orbital period; thus there were 12 station-keeping segments. Errors were added to the formation control maneuvers every third segment while the formation control maneuvers were perfectly executed every first and second segment. During every third segment, the helix should drift away from the nominal halo orbit. If the station-keeping algorithm is properly formulated, all of the drift in the helix will be corrected in the following segment where the formation maneuvers are perfectly executed.

The algorithm was verified, shown in Figure 6 and Figure 7. Figure 6 shows how the black helix is first aligned with the nominal orbit when no errors are added. Then, the formation drifts from the nominal halo orbit, indicated by the dashed red line, when errors are added to the simulation. A SKM is implemented, signified by the black circles, and the formation is corrected back to the nominal halo orbit during segments where the formation control maneuvers are executed perfectly. Figure 7 shows corresponding plots of the magnitude of the distance of the helix center to the nominal halo orbit and the magnitude of the maneuvers, respectively. In Figure 7, the maneuver sequence is illustrated by the red line, where the spikes correspond to the SKMs, and the nearly constant horizontal line near zero cm/s corresponds to the helix maneuvers. It should be noted that the SKMs at days 15 and 30 are no larger than the helix maneuvers. This is due to the fact that no errors are added in this segment, so there is little drift to correct; only drift due to numerical errors is corrected and those corresponding SKMs are no larger than helical maneuvers. However, when errors are added to the formation control maneuvers between 30 and 45 days, the magnitude of the distance of the center of the helix to the nominal orbit increases. A large SKM is performed at 45 days to correct the helix drift, and immediately the helix starts on a correction trajectory back to the nominal halo orbit. At 60 days, the position of the helix is perfectly aligned with the nominal halo orbit. However, the velocity at this state is incorrect; the differential corrector scheme only targets the position at the final state. Therefore, another large maneuver is performed at 60 days to correct the difference in velocity between the nominal halo orbit and the drifted helix. After the SKM is executed perfectly at 60 days, the helix follows the nominal trajectory out to 75 days since no errors are added in this segment. This third segment error addition is continued over the course of one orbital period. After each error addition segment, the helix is perfectly corrected, providing assurance that the algorithm can correctly target back to the nominal orbit. It should also be noted that for this verification simulation, the errors added to the formation control maneuvers were several orders of magnitude larger than the errors that will be added to determine realistic SKM budgets. This was done not only to graphically illustrate the helix drift previously discussed, but also to demonstrate the capabilities of the software to correct significantly large formation drift.

IV. Results

Results have been generated to determine the yearly estimated budget of the proposed TPF-I mission and the optimal station-keeping period. Specific position and velocity OD errors have been used to generate the data. The position OD errors used in the study are drawn from Gaussian distributions with zero mean and standard deviations of 10 km, 1 km, 0.1 km, 10 m and 1 m. Velocity OD errors are drawn from Gaussian
Figure 6. Every third segment, the black helix drifts from the nominal halo orbit (red), but the algorithm provides a correction trajectory back to the nominal. The red stars are the SKM target points, and the black circles represent points where the SKMs are executed.

Figure 7. Magnitude of the distance of the drifted helix center to the nominal halo orbit (left), and the corresponding maneuver sequence (right).

distributions with zero mean and standard deviations of 1 cm/s, 0.1 cm/s and 0.01 cm/s. Results have been classified into two cases based on the formation control maneuver errors.

IV.A. Case 1 Results

For Case 1, one hundred maneuvers are executed each day to create a helix with a diameter of 30 m about the nominal halo orbit. Formation maneuver errors are added to each maneuver and are drawn from Gaussian distributions with zero mean and standard deviations of 5%, 5%, and 2% of the components of the maneuvers in the $x$-, $y$-, and $z$-directions respectively. Station-keeping maneuvers are executed every 60 days and each SKM targeted the position on the nominal halo orbit 60 days in advance. Ten simulations were conducted and the results were averaged and presented in Figure 8. Table 2 presents the full results comparison for the simulations conducted under the conditions of Case 1. In these simulations, the largest errors added were position OD errors with standard deviations of 10 km and velocity OD errors with standard deviations of 1 cm/s. This will be known as Case 1A. It can be seen from Figure 8 that for Case 1A, the average yearly SKM budget is approximately 70 cm/s per satellite. The largest and smallest budgets for Case 1A were 108 cm/s per satellite and 36 cm/s per satellite respectively and the standard deviation of the results was 22 cm/s.

Figure 8 shows that an increase in velocity OD accuracy is more beneficial than an increase in the position
OD accuracy. When the standard deviation of the velocity OD errors improves from 1 cm/s to 0.1 cm/s, the yearly SKM budget drops by approximately 40 cm/s per satellite. A less noticeable improvement is seen when the standard deviation of the velocity OD errors improves by another order of magnitude. When the standard deviation of the position OD errors improves from 10 km to 1 km, the yearly SKM budget decreases by approximately 20 cm/s per satellite. However, as the position OD error standard deviation improves, little improvement is seen. Further benefit may only be obtained by reducing the maneuver executing errors.

The position of the center of the helix is tracked at all times during one of the Case 1A simulations and is plotted in Figure 9. It can be seen that the formation drifts from the nominal halo orbit by as much as 350 km during the course of the simulation, however when a maneuver is executed, the distance from the formation to the nominal orbit decreases and some of the drift is corrected. However, before the drift is entirely corrected back to the nominal orbit, the formation drifts again due to the accumulation of errors in the formation control maneuvers and imperfect execution of the SKM. Figure 9 shows the magnitudes of the maneuvers over simulation. It should be noted that the largest SKM comes at 120 days and corresponds to the point in the mission where the formation has drifted farthest from the nominal orbit. The average SKM for this simulation is approximately 20 cm/s. Although the SKMs on this order of magnitude may seem very small, they are relatively large compared to the formation control maneuvers. On average, each of the formation control maneuvers is only 60 µm/s. These maneuvers are so small because there is very little distance to travel from one helix vertex to the next. However, there are 100 of these maneuvers executed daily, and the accumulation of the formation control maneuvers actually yields a yearly budget of around 2 m/s per satellite, which is more than the SKM budget.

![Figure 8](image-url)

**Figure 8.** SKM Budget for 2 orbit periods under the conditions of Case 1 - Standard deviations of formation control maneuver errors are 5%, 5%, and 2% of the maneuver components in the x-, y-, and z-directions respectively.
Table 2. A comparison of the results for Case 1.

<table>
<thead>
<tr>
<th>Velocity OD Error</th>
<th>Statistics</th>
<th>Position OD Error Standard Deviation</th>
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<tbody>
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<td></td>
<td></td>
<td>10 km</td>
</tr>
<tr>
<td>1 cm/s</td>
<td>Average SKM Budget for 2 Orbital Periods, cm/s</td>
<td>66.246</td>
</tr>
<tr>
<td></td>
<td>Largest SKM Budget, cm/s</td>
<td>108.274</td>
</tr>
<tr>
<td></td>
<td>Smallest SKM Budget, cm/s</td>
<td>46.190</td>
</tr>
<tr>
<td>0.1 cm/s</td>
<td>Average SKM Budget for 2 Orbital Periods, cm/s</td>
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<tr>
<td></td>
<td>Smallest SKM Budget, cm/s</td>
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</table>

IV.B. Case 2 Results

Case 2 simulations are similar to those for Case 1; one hundred maneuvers are executed each day to create a helix with a diameter of 30 m about the nominal halo orbit. However, the magnitudes of the formation control maneuvers are smaller; errors are drawn from Gaussian distributions with zero mean and standard deviations of 1% of the maneuver components in the x-, y-, and z-directions. Station-keeping maneuvers were executed every 60 days and each SKM targeted the position on the nominal halo orbit 60 days in advance. The results of 10 simulations were averaged and are presented in Figure 10. Comparing Figure 8 and Figure 10 shows nearly identical trends with a slight improvement in numerical results, attributed to the improvement in the execution of the formation control maneuvers. Case 2A will be identified similarly to Case 1A above, where the largest combination of position and velocity OD errors are added to the simulation. It can be seen from the figures that when the standard deviation of the errors in the formation control maneuvers improves to 1% in the x-, y-, and z-directions, the yearly SKM budget only decreases by 5 cm/s per satellite. Therefore, it appears to be more beneficial to improve the OD errors rather than to improve the accuracy.
of the thrusters to execute formation control maneuvers. Table 3 presents a full results comparison for the simulations conducted in Case 2.

![Figure 10. SKM Budget for 2 orbit periods under the conditions of Case 2 - Standard deviations of formation control maneuver errors are 1% of the maneuver components in the \( x \), \( y \), and \( z \)-directions respectively.](image)

### Table 3. A comparison of the results for Case 2.

<table>
<thead>
<tr>
<th>Velocity OD Error</th>
<th>Statistics</th>
<th>Position OD Error Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10 km</td>
</tr>
<tr>
<td>1 cm/s</td>
<td>Average SKM Budget for 2 Orbital Periods, cm/s</td>
<td>60.639</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation, cm/s</td>
<td>22.352</td>
</tr>
<tr>
<td></td>
<td>Largest SKM Budget, cm/s</td>
<td>100.615</td>
</tr>
<tr>
<td></td>
<td>Smallest SKM Budget, cm/s</td>
<td>24.187</td>
</tr>
<tr>
<td>0.1 cm/s</td>
<td>Average SKM Budget for 2 Orbital Periods, cm/s</td>
<td>30.464</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation, cm/s</td>
<td>12.001</td>
</tr>
<tr>
<td></td>
<td>Largest SKM Budget, cm/s</td>
<td>49.539</td>
</tr>
<tr>
<td></td>
<td>Smallest SKM Budget, cm/s</td>
<td>6.190</td>
</tr>
<tr>
<td>0.01 cm/s</td>
<td>Average SKM Budget for 2 Orbital Periods, cm/s</td>
<td>28.753</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation, cm/s</td>
<td>10.972</td>
</tr>
<tr>
<td></td>
<td>Largest SKM Budget, cm/s</td>
<td>45.070</td>
</tr>
<tr>
<td></td>
<td>Smallest SKM Budget, cm/s</td>
<td>7.797</td>
</tr>
</tbody>
</table>
IV.C. Budget for the Collect vs. the Combiner

In all of the results presented thus far, only one satellite, the combiner, was used to generate the helical trajectories and determine the overall SKM budget. It was assumed that the budget for the combiner would be very similar to the budget of the collectors. Simulations were conducted to verify this assumption. The outer collector satellite was used under the same conditions as described in Case 1A and the SKM budget for 2 orbital periods was determined from the average of 10 simulations. As shown in Table 2 the budget for the combiner satellite for 2 orbital periods under the conditions of Case 1A was approximately 66 cm/s. For the same conditions (albeit different sets of random errors), the budget for the outer collector for 2 orbital periods was approximately 67 cm/s. These budgets are very similar, considering that they are the average of 10 simulations using different sets of random numbers to generate the errors. It is expected that if more simulations were averaged to generate the budget, the results would be more similar.

V. Frequency of SKMs

Simulations have been conducted to determine the optimal time between executing SKMs. The time between SKMs will be referred to as the station-keeping period. The station-keeping period has been varied from 40 to 80 days in 10-day increments. Position and velocity OD errors are drawn from Gaussian distributions with standard deviations of 10 km and 1 cm/s respectively. Formation maneuver errors were drawn from Gaussian distributions based on the criteria given in Case 1. Ten simulations were conducted over the time interval of two orbital periods. The data generated from these ten simulations were averaged and the results are presented in Figure 11. Figure 11 shows that a station-keeping period of 60 days is nearly the optimal result. There is little benefit gained by decreasing the station-keeping period to 40 or 50 days. However, increasing the station-keeping period from 60 to 70 days shows a two-fold increase in the yearly SKM budget. If the station-keeping period is increased from 60 days to 80 days, the yearly SKM budget is approximately seven times greater. Furthermore, on average for a station-keeping period of 60 days, the center of the helix trajectory drifts 150 km from the nominal orbit before an SKM is implemented to correct the drift. However, for the station-keeping period of 80 days, the center of the helix drifts as far as 3600 km before SKM execution. A sixty-day station-keeping period, therefore, appears to be the ideal time to execute SKMs before the errors accumulate and the formations drifts so far from the orbit that larger SKMs are necessary to correct the trajectory.

Figure 11. Yearly station-keeping budgets for various station-keeping periods.
VI. Helix Plane Changes

As stated in the assumptions, the proposed TPF-I formation rotates in a plane once every 24 hours to face a target star. In the previous simulations, the target star has always been assumed to face only in the positive $x$-direction, with no components of the target plane in the $y$- or $z$-directions. However, in the mission, the formation must change planes to face different stars for observation. Several simulations were conducted to determine the changes to the SKM budget if the formation were to rotate. The simulations were run under the same conditions as Case 1A, which is described in Section IV.A. The formation was allowed to change planes every 48 hours by performing a plane-change maneuver (PCM). The formation would focus on one star for 24 hours, and then would be allowed to changes planes over the course of the next 24 hours to allow for correct positioning of the spacecraft before new observational data are taken. The rotation was limited so the angle between successive plane changes was less than 25 degrees (to minimize the maneuver magnitude necessary to change planes), and the target plane always had positive components in the $x$-direction. The results from ten simulations were averaged and compared to the results where the formation did not change planes. This comparison is show in Table 4.

<table>
<thead>
<tr>
<th></th>
<th>Non-plane changing Helix</th>
<th>Plane Changing Helix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average SKM Budget for 2 Orbital Periods, cm/s</td>
<td>66.246</td>
<td>70.384</td>
</tr>
<tr>
<td>Average Maximum Drift from Nominal Orbit, km</td>
<td>330.40</td>
<td>343.97</td>
</tr>
<tr>
<td>Average Total Budget of all maneuvers, cm/s</td>
<td>242.20</td>
<td>543.61</td>
</tr>
</tbody>
</table>

The results show that introducing plane changes to the rotating helix produces a budget that is, on average, approximately 4 cm/s greater for two orbital periods than the budget of a helix which does not change planes. The magnitude of each SKM is generally closely related to how far the satellite drifts from the nominal orbit, i.e. how much error in distance the maneuver must correct. The drift of the satellite from the nominal orbit was tracked over time and the maximum distance the spacecraft drifted during each of the simulations was averaged and presented in the second column of Table 4. The results show that the helix which changes planes does not drift much farther from the nominal orbit than its non-plane changing counterpart. Finally, the average total budget — the sum of the formation maneuvers, the SKMs, and the PCMs (if applicable) is presented in the third row. It can be seen that the total budget for the helix that changes planes is more than double of the helix that does not change plane. This large increase may be further explained by looking at a single simulation, described below.

Two simulations were conducted, one with and one without helix plane changes. The simulations were conducted with the error conditions specified in Case 1A and the helix performed a PCM every 2 days. Both simulations used the same set of random errors added to the maneuvers to enable a more direct comparison between the results.

The position of the center of the helix was tracked during the simulation and the magnitude of the distance from the helix center to the nominal halo orbit was plotted and is presented in Figure 12. It can be seen that the drift is nearly identical in shape, although the peaks in distance are higher for the case where the helix changes planes. This plot is closely related to the magnitude of the SKMs. As stated before, the magnitude of the SKM generally corresponds to the amount of drift that must be corrected. This can be seen in Figure 13. Figure 13 presents the maneuver sequences of the two simulations. The largest spikes in the maneuver magnitudes correspond to the SKMs, which are performed every 60 days. Comparison of Figure 12 and Figure 13 shows that the largest SKM, performed at 60 days, corresponds to the largest drift of the spacecraft from the nominal orbit. The plot on the right of Figure 13 also illustrates the magnitude of the frequent PCMs. It can be seen that most of the PCMs are on the order of 1 cm/s. Table 5 presents a full numerical breakdown of the total budget. It can be seen that the total SKM budgets are quite similar, differing by less than 5 cm/s for two orbital periods. This is consistent with the results from the
average of 10 simulations, presented in Table 4. The average PCM is less than 1 cm/s; however, over two orbital periods, approximately 180 PCMs are performed. These PCMs accumulate for a total PCM budget of 144 cm/s, which is larger than the total SKM budget, despite the fact that the average SKM is over 10 times larger than the average PCM. Furthermore, the budget for the formation maneuvers increases for the simulation with helix plane changes. The PCMs occur every 48 hours, which targets only the position of the next vertex of the 100-gon on a new plane. The velocity difference between the vertex of helix on one plane and the vertex of the helix on a new plane is much more different than the velocity difference between vertices of the 100-gon on the same plane. Therefore, the formation maneuvers which occur directly after the PCM are larger than average formation maneuvers because they have to correct the velocity differences after the plane change. These factors contribute to the larger overall budget. However, the important factor to note is that the SKM budget does not increase greatly with the addition of plane changes to the simulation.

![Figure 12. Comparison of the drift of the spacecraft from the nominal orbit during two orbital periods: No helix plane-changes (left), Helix with plane changes (right).](image)

![Figure 13. Comparison of the maneuver magnitude sequence: No helix plane-changes (left), Helix with plane changes (right).](image)
Table 5. Budget and Drift results for one simulation of 2 Orbital Periods with and without helix plane changes

<table>
<thead>
<tr>
<th></th>
<th>Non-plane changing Helix</th>
<th>Plane Changing Helix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average SKM cm/s</td>
<td>15.44</td>
<td>16.43</td>
</tr>
<tr>
<td>Average PCM, cm/s</td>
<td>N/A</td>
<td>0.84</td>
</tr>
<tr>
<td>SKM Budget, cm/s</td>
<td>77.72</td>
<td>82.17</td>
</tr>
<tr>
<td>PCM Budget, cm/s</td>
<td>N/A</td>
<td>144.11</td>
</tr>
<tr>
<td>Formation Maneuver Budget, cm/s</td>
<td>175.84</td>
<td>318.84</td>
</tr>
<tr>
<td>Total Budget, cm/s</td>
<td>253.56</td>
<td>545.12</td>
</tr>
<tr>
<td>Maximum Drift from Nominal Orbit, km</td>
<td>321.02</td>
<td>333.35</td>
</tr>
</tbody>
</table>

VII. Conclusions

A station-keeping maneuver strategy has been developed for the proposed TPF-I mission with the L2 option. We have shown that it is possible to create a seamless helical trajectory that will closely follow a nominal halo orbit by the execution of SKMs. We have further shown that this method yields a very reasonable yearly SKM budget for the mission. The simulation with the largest errors added to the maneuvers produced an average yearly SKM budget of 70 cm/s per satellite. In addition, we have found that this mission can be accomplished by executing SKMs every 60 days. The assumption that the budget of the collector will accurately represent the budget of any of the satellites in the formation was proven, as the SKM budget changed little in comparing the budgets of two different satellites. The addition of plane changes to the simulation increased the SKM budget by approximately 5 cm/s per satellite for two orbital periods. Future work will examine the differential corrector targeting scheme and extend it to study the effects of targeting both position and velocity. This work may also be extended to other missions at libration point orbits to help reduce SKM budgets in the future.

VIII. Acknowledgments

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References