LINKED, AUTONOMOUS, INTERPLANETARY SATELLITE ORBIT NAVIGATION (LiAISON) IN LUNAR HALO ORBITS

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In this study, simulated Satellite-to-Satellite Tracking (SST) measurements for spacecraft in lunar halo orbits were used to investigate the feasibility of Liaison Navigation. The lunar halo orbits were generated using the circular restricted three-body problem. Through covariance analysis, it was found that the position and velocity vectors for all participating spacecraft were observable using Liaison Navigation methods (using only crosslink range without Earth-based tracking). The accuracy of the Liaison Navigation orbit estimates was compared for various constellation geometries. The accuracy was better for orbits that did not remain near the lunar orbit plane and for orbits with shorter periods. The orbit accuracy decreased as the separation distance between the spacecraft decreased, meaning that close formations could not effectively use Liaison Navigation. A range bias was successfully included as an estimated parameter and its effect on the orbit accuracy was shown. The effect on the orbit accuracy was also shown for simulations that included constant force model errors. It was found that Liaison Navigation performed very well for certain constellation geometries. Monte Carlo analysis was used to investigate the validity of the covariance. It was found that the covariance was slightly optimistic, probably due to correlation between the two satellite state estimates. With the type of current on-board hardware available, it should be possible to achieve $1\sigma$ orbit errors on the order of 10 m using Liaison Navigation in lunar halo orbits.

INTRODUCTION

Most observations used to determine the orbits of interplanetary spacecraft are generated using Earth-based tracking sensors. Autonomous orbit determination is performed with measurements obtained using only the equipment on board orbiting spacecraft. If interplanetary spacecraft could use autonomous orbit determination, these missions would be less reliant on Earth-based trackers such as the Deep Space Network (DSN). A new type of autonomous orbit determination was proposed called Linked, Autonomous, Interplanetary Satellite Orbit Navigation (LiAISON).¹ LiAISON, hereafter referred to as Liaison Navigation, is a method where two or more spacecraft can determine their orbits using only Satellite-to-Satellite tracking (SST) data, which are scalar observations taken along the line-of-sight vector between two spacecraft such as crosslink range and range-rate. In the two-body problem, SST by itself cannot be used alone to estimate the absolute positions of the spacecraft involved. However, SST does provide information on the size, shape, and relative orientation of the orbits. Suppose an orbit exists with a unique size and shape and it has only one possible absolute orientation in space. If a spacecraft were following this unique orbit and SST observations could be used to correctly determine the size and shape of the orbit, it follows that since there is only one possible orientation for an orbit of that size and shape, the absolute position of the spacecraft on the unique trajectory could be known and no ground based tracking would be needed. Since SST also provides information on the relative orientation of the trajectories of the other spacecraft involved, the absolute positions of all the other spacecraft participating in the SST observations would be known as well. Thus, if one spacecraft is in a unique, or locally unique, trajectory, SST observations generated

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between it and another spacecraft could be used to estimate the absolute positions of both. That is the definition of Liaison Navigation: using only SST to determine the orbits of two or more participating spacecraft when one of them is in an orbit of a locally unique size and shape. It was shown that the third-body perturbation of the Moon provides enough asymmetry in the acceleration field that locally unique orbits exist as long as they are not restricted to the plane of the Moon’s orbit. Liaison Navigation was shown to be more likely to succeed in the regions where the gravitational forces of the Earth and the Moon were similar in magnitude. Orbits around the Lunar L₁ and L₂ libration points are ideal for testing Liaison Navigation since those orbits are locally unique and always remain in regions where the asymmetry of the third body perturbation is very strong.

**LIAISON NAVIGATION METHODS**

Orbit determination attempts to estimate the initial conditions of the nonlinear system of differential equations describing spacecraft motion using measurements that are nonlinear with respect to the state. This is done by linearizing about a reference orbit that is close to the actual orbit. Instead of estimating the initial state with the measurements, the error in the initial state is estimated using the observation residuals, which are the observed measurements minus those computed from the reference solution. Liaison Navigation can be conducted using any type of orbit determination method. However, in this study the two different techniques used were the batch processor and the Kalman Filter. A summary of these techniques comes from Tapley, et al., and uses their notation. For illustrative purposes, the equations are shown for two spacecraft, although the number of spacecraft participating can be larger.

The spacecraft state consists of three position and velocity components for each spacecraft. For two spacecraft, it would be

\[
\mathbf{X} = \begin{bmatrix} x_1 & y_1 & z_1 & \dot{x}_1 & \dot{y}_1 & \dot{z}_1 & x_2 & y_2 & z_2 & \dot{x}_2 & \dot{y}_2 & \dot{z}_2 \end{bmatrix}^T,
\]

where the subscripts denote the spacecraft number. The equations of motion are written

\[
\dot{\mathbf{X}} = F(\mathbf{X}, t), \quad \mathbf{X}_i \equiv \mathbf{X}(t_i).
\]

An observation at time \(t_i\) is denoted by \(\mathbf{Y}_i\), and the state deviation and observation residuals are written

\[
\mathbf{x}(t) = \mathbf{X}(t) - \mathbf{X}^*(t), \quad \mathbf{y}(t) = \mathbf{Y}(t) - \mathbf{Y}^*(t),
\]

where \(\mathbf{X}^*\) is the reference solution, or the “best guess” orbit, and \(\mathbf{Y}^*\) is an observation computed using the reference solution. The observations can be related to the state with the \(\hat{H}\) matrix, which is

\[
\hat{H}_i = \left[ \frac{\partial G}{\partial \mathbf{X}} \right]_i^*.
\]

\(G\) is a function that computes observations from a state vector in the following way:

\[
\mathbf{Y}_i = G(\mathbf{X}_i, t_i) + \epsilon_i.
\]

\(\epsilon_i\) is the observation error. If crosslink range, \(\rho\), and crosslink range rate, \(\dot{\rho}\), are the observations types then \(\hat{H}_i\) is of the form:

\[
\tilde{H}_i = \begin{bmatrix} \frac{\partial \rho}{\partial x_1} & \frac{\partial \rho}{\partial y_1} & \frac{\partial \rho}{\partial z_1} & \frac{\partial \rho}{\partial \dot{x}_1} & \frac{\partial \rho}{\partial \dot{y}_1} & \frac{\partial \rho}{\partial \dot{z}_1} & \frac{\partial \rho}{\partial x_2} & \frac{\partial \rho}{\partial y_2} & \frac{\partial \rho}{\partial z_2} & \frac{\partial \rho}{\partial \dot{x}_2} & \frac{\partial \rho}{\partial \dot{y}_2} & \frac{\partial \rho}{\partial \dot{z}_2} \\
\frac{\partial \dot{\rho}}{\partial x_1} & \frac{\partial \dot{\rho}}{\partial y_1} & \frac{\partial \dot{\rho}}{\partial z_1} & \frac{\partial \dot{\rho}}{\partial \dot{x}_1} & \frac{\partial \dot{\rho}}{\partial \dot{y}_1} & \frac{\partial \dot{\rho}}{\partial \dot{z}_1} & \frac{\partial \dot{\rho}}{\partial x_2} & \frac{\partial \dot{\rho}}{\partial y_2} & \frac{\partial \dot{\rho}}{\partial z_2} & \frac{\partial \dot{\rho}}{\partial \dot{x}_2} & \frac{\partial \dot{\rho}}{\partial \dot{y}_2} & \frac{\partial \dot{\rho}}{\partial \dot{z}_2} \end{bmatrix}_i
\]

State deviations at time \(t\) may be mapped back to the initial epoch, \(t_k\), with the state transition matrix \(\Phi\) in the following way:

\[
\mathbf{x}(t) = \Phi(t, t_k)\mathbf{x}_k.
\]
The state transition matrix is integrated along with the state using the following relation:

\[
\dot{\Phi}(t, t_k) = A(t)\Phi(t, t_k)
\]

where \( A(t) = \frac{\partial F(X^*, t)}{\partial X(t)} \) and is of the form:

\[
A = \begin{bmatrix}
\frac{\partial \hat{x}_1}{\partial x_1} & \frac{\partial \hat{x}_1}{\partial y_1} & \frac{\partial \hat{x}_1}{\partial z_1} & \cdots & \frac{\partial \hat{y}_1}{\partial z_2} \\
\frac{\partial \hat{y}_1}{\partial x_1} & \frac{\partial \hat{y}_1}{\partial y_1} & \frac{\partial \hat{y}_1}{\partial z_1} & \cdots & \frac{\partial \hat{z}_1}{\partial z_2} \\
\frac{\partial \hat{z}_1}{\partial x_1} & \frac{\partial \hat{z}_1}{\partial y_1} & \frac{\partial \hat{z}_1}{\partial z_1} & \cdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial \hat{z}_2}{\partial x_1} & \cdots & \frac{\partial \hat{z}_2}{\partial x_1} & \cdots & \frac{\partial \hat{z}_2}{\partial z_2}
\end{bmatrix}
\]

Relating an observation at time \( t_i \) back to the state initial epoch \( t_k \) is done with the \( H_i \) matrix:

\[
H_i = \hat{H}_i\Phi(t_i, t_k)
\]

So \( H_i \) is the observation-state relationship at \( t_i \) mapped back to the initial epoch \( t_k \). If there are \( \ell \) observations, all of them can be combined into a vector, and all of the \( H_i \) matrices into a larger matrix so that

\[
y = H\mathbf{x}(t_k) + \epsilon \quad \text{where} \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_\ell \end{bmatrix} \quad \text{and} \quad H = \begin{bmatrix} H_1 \\ \vdots \\ H_\ell \end{bmatrix} \quad \text{and} \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_\ell \end{bmatrix}
\]

A least squares solution for \( \mathbf{x}(t_k) \) is computed with the equation

\[
\hat{x} = (H^TH)^{-1}H^Ty.
\]

If it is assumed that the observation errors can be modeled as white noise, and the standard deviation in the observation noise for observation type \( i \) is denoted \( \sigma_i \), a weighting matrix \( W \) can be used to weight the observations such that

\[
\hat{x} = (H^TWH)^{-1}H^Ty,
\]

where

\[
W = \begin{bmatrix}
\sigma_1^2 & & \\
& \ddots & \\
& & \sigma_n^2
\end{bmatrix}^{-1}, \quad \text{and} \quad n \text{ is the number of observation types.}
\]

The matrices \( H^TWH \) and \( H^TWy \) can be accumulated one observation at a time:

\[
H^TWH = \sum_{i=1}^{\ell} H_i^TWH_i \quad \text{and} \quad H^TWy = \sum_{i=1}^{\ell} H_i^TWy_i.
\]

\( H^TWH \) is also called the information matrix, \( \Lambda \). The variance-covariance matrix is the inverse of the information matrix:

\[
P = \Lambda^{-1}.
\]

The covariance matrix can be used to compute the variances of the state parameters as well as the correlations between state parameters. The \( 3 \times 3 \) position portions of the covariance matrix can also be used to plot a 3D error
ellipsoid for each satellite. The eigenvalues of the $3 \times 3$ position portions of the covariance matrix correspond to the lengths of the semimajor axes of the ellipsoid, and the matrix of eigenvectors is a rotation from the coordinate frame in use to the principle axes coordinate frame. As long as several assumptions are correct, there is a 97% statistical probability that the true initial state lies within the 3D, $3\sigma$ error ellipsoid.

If a covariance matrix for the a priori estimate of the state, $P_0$, is known, it can be included in the computations using the equation

$$
\Lambda = P_0^{-1} + \sum_{i=1}^{\ell} H_i^T W H_i.
$$

When using SST without any Earth-based tracking for orbits in the two-body problem, the initial state is not entirely observable. The entire state is observable only if $\Lambda$ is positive definite. Numerically, $\Lambda$ can be considered non-positive definite if its condition number is greater than $10^{16}$. To test whether Liaison Navigation is feasible, one of the most important steps is verifying that $\Lambda$ is positive definite without Earth-based tracking. To check for true observability, the a priori covariance should not be used in the equation for computing $\Lambda$, since $P_0$ may artificially make the state observable when it should not be. When accumulating $\Lambda$ during Liaison Navigation, the differences in the state transition matrices of the spacecraft, when one is in a unique orbit, make $\Lambda$ positive definite. The $H_i$ matrix would tend to make the rows of $\Lambda$ dependent, since many of the partials used to create $H_i$ are equal in magnitude and opposite in sign, like the partials $\frac{\partial P}{\partial x_1} = -\frac{\partial P}{\partial x_2}$.

The batch processor is very useful at determining observability, but is not capable of giving an estimate of how the covariance would change when processing observations in real time. By modifying the batch processor so that the observations can be processed sequentially, the resulting filter can be used to show the number of observations and how long it takes for the processor to converge on an orbit solution. The sequential filter used for these tests was the conventional Kalman filter. The Kalman filter requires an a priori covariance, $P_0$, so it cannot be used to verify total observability, but can show how the covariance changes in real time. The Kalman filter works by propagating the state deviation and the covariance at time $k - 1$ to the next observation epoch, $k$, using the “time update” equations

$$
\bar{x}_k = \Phi(t_k, t_{k-1}) \bar{x}_{k-1} - 1 \quad \text{and} \quad \bar{P}_k = \Phi(t_k, t_{k-1}) P_{k-1} \Phi^T(t_k, t_{k-1}).
$$

The state deviation and covariance are updated with the observations at time $k$ using the “measurement update” equations

$$
K_k = \bar{P}_k \bar{H}_k^T \left[ \bar{H}_k \bar{P}_k \bar{H}_k^T + W^{-1} \right]^{-1},
$$

$$
\bar{x}_k = \bar{x}_{k-1} + K_k [y_k - \bar{H}_k \bar{x}_{k-1}],
$$

$$
P_k = \left[ I - K_k \bar{H}_k \right] \bar{P}_k.
$$

After creating estimates of the state vector, it is very useful to be able to compare the accuracy of the orbit estimates. In this study, the method used to compare the accuracy of orbit estimates is to propagate the batch covariance matrix over the entire fit span with the relation

$$
P_t = \Phi(t_i, t_k) P_k \Phi^T(t_i, t_k).
$$

Then compute the length of the largest axis of the 3D, $3\sigma$ error ellipsoid for the spacecraft, which will be called $\beta_i$ at time $t_i$.

$$
\beta_i = 3 \max \left( \sqrt{\lambda_j} \right) \quad \text{where } \lambda_j \text{ for } j = 1, \ldots, 3 \text{ are the eigenvalues of } [P_t]_{3 \times 3}.
$$

The average value of $\beta_i$ over the entire fit span will be called $\bar{\beta}$:

$$
\bar{\beta} = \frac{\sum_{i=1}^{n} \beta_i}{n} \text{ where there are } n \text{ values of } \beta_i \text{ in the fit span.}
$$
If there are multiple satellites, the values of $\beta$ for all of the satellites can be averaged to produce a metric called $\beta_{ave}$ that gives an indication of the overall navigation accuracy for the constellation for those particular conditions.

THE CIRCULAR RESTRICTED THREE-BODY PROBLEM

The halo orbits used to simulate Liaison Navigation were all generated using the dynamic model called the circular restricted three-body problem (CRTBP). In this model, there are two massive bodies in orbit about their mutual barycenter, which is depicted in Figure 1. To simplify the model, each body orbits the barycenter in the same plane in perfectly circular orbits. A spacecraft with infinitesimal mass experiences forces due to the gravitational influence of both bodies simultaneously, and the two bodies are approximated as point masses. The more massive body is labeled $P_1$ and the other is $P_2$. The coordinate frame has its origin at the barycenter and rotates with the two bodies so that $P_1$ and $P_2$ are always on the x-axis, with the positive x-direction going from $P_1$ to $P_2$. The positive y-axis is parallel to the velocity vector of $P_2$. The units are nondimensionalized so that the mass of $P_2$ is defined to be $\mu$ and the mass of $P_1$ is $1 - \mu$. The distance along the x-axis from the origin to $P_1$ is $-\mu$ and from the origin to $P_2$ is $1 - \mu$. The time unit is defined such that $P_2$ orbits around $P_1$ in $2\pi$ time units.

![Figure 1 Diagram of the Circular Restricted Three-Body Problem with a rotating, nondimensional coordinate frame.](image)

The equations of motion for the CRTBP are

\[
\begin{align*}
\dot{x} - 2\dot{y} &= x - (1 - \mu) \frac{x + \mu}{r_1^3} - \mu \frac{x + \mu - 1}{r_2^3} \\
\dot{y} + 2\dot{z} &= \left(1 - \frac{1 - \mu}{r_1^3} - \frac{\mu}{r_2^3}\right) y \\
\dot{z} &= \left(\frac{\mu - 1}{r_1^3} - \frac{\mu}{r_2^3}\right) z
\end{align*}
\]

where $r_1 = \sqrt{(x + \mu)^2 + y^2 + z^2}$ and $r_2 = \sqrt{(x + \mu - 1)^2 + y^2 + z^2}$.

Five equilibrium points exist in the CRTBP when using a rotating coordinate frame. A spacecraft placed on an equilibrium point with zero velocity will remain at that equilibrium point indefinitely. These points are called Lagrange points, and they are shown for the Earth-Moon system in Figure 2. Three of the equilibrium points are
Liaison Navigation Orbit Geometry

To evaluate the effectiveness of Liaison Navigation, a group of test orbits was created. These test orbits were designed so that matching various test orbits with each other could simulate a wide range of two-orbit combinations (constellations). At least the first of the orbits in any constellation would have to be a halo orbit and it will be called “Orbit 1,” while the second orbit could be of any type and will be named “Orbit 2.” The computation of the halo orbits was performed using the technique explained by Howell. Beginning at the x-z plane with velocity only in the y-direction, an orbit is propagated using the CRTBP equations of motion. When the orbit crosses the x-z plane again, the state transition matrix is used in a differential corrector to adjust the initial conditions so that the velocity at the second x-z plane crossing is perpendicular to the x-z plane. This is done in an iterative manner until the velocity at the second crossing of the x-z plane is perpendicular to within a certain tolerance.

Families of halo orbits were generated around all three of the collinear Lagrange points in the Earth-Moon system (LL₁, LL₂, and LL₃). For each family of halo orbits, a certain number were selected as test orbits and ephemeris files were generated for each one. Figures 3, 4, and 5 show the distribution of the initial conditions of the test halo orbits. The halo ephemerides were integrated using a fixed-step, fourth-order Runge-Kutta method with a time step of $2 \times 10^{-4}$ time units. A set of lunar orbits and Earth orbits were generated for testing as well. Table 1 lists each of the test orbits and their characteristics. The orbit elements in the table are defined for the two-body problem, so they were only used to generate the initial conditions for the orbits. The orbits themselves were propagated using the equations of motion for the three-body problem in the same reference frame as the halo orbits. The lunar and Earth ephemerides were integrated using a time step of $5 \times 10^{-5}$ time units. The Earth orbits were meant to be similar to an orbit in the GPS constellation and a geostationary orbit.

The two spacecraft could be placed in the different test orbits in several different ways. When both spacecraft are in the same halo orbit, the constellation is called “Halo-Halo.” When the spacecraft are in different halo orbits, the constellation is called “Halo-Halo,” and finally, if the spacecraft are in a halo and a lunar or Earth-centered orbit, the constellation is called “Halo-Moon” or “Halo-Earth.”
Figure 3  Initial conditions and names of LL₁ test orbits.

Figure 4  Initial conditions and names of LL₂ test orbits.
Table 1 PARAMETERS OF THE LUNAR- AND EARTH-CENTERED TEST ORBITS.

<table>
<thead>
<tr>
<th>Orbit Name</th>
<th>$i$ (deg)</th>
<th>$h_p$ (km)</th>
<th>$h_a$ (km)</th>
<th>$\Omega$ (deg)</th>
<th>period (days)</th>
</tr>
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<td></td>
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<td>100</td>
<td>100</td>
<td>270</td>
<td>0.082</td>
</tr>
<tr>
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<td>100</td>
<td>180</td>
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<tr>
<td>Lb</td>
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<td>270</td>
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<tr>
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</tr>
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Note: For all of the orbits, the true anomaly and argument of perigee were 0 degrees.

LIAISON NAVIGATION TEST METHODS

The first Liaison Navigation tests were conducted using two spacecraft that tracked each other continuously, unless the Earth or the Moon blocked the line of sight (LOS) vector. Range observations were generated from the ephemeris files every 0.001 time units ($\approx 375$ seconds). These observations were used to generate the $\dot{H}_t$
matrices. The state transition matrices were obtained from the ephemeris files for each orbit. The information matrix, $H^TWH$, was computed without computing $H^T Wy$ and the covariance was used to compute navigation error metrics. The range observations had an added white noise component with a standard deviation of 1 m.

When placing the spacecraft in halo orbits, it became convenient to define a phase angle, $\tau$. $\tau$ is a non-geometric angle describing the positions of the spacecraft in the halos in a manner similar to mean anomaly in that the phase angle has a constant time derivative. A phase angle of zero occurs when the spacecraft is on the halo orbit where it intersects the x-z plane traveling in the positive y-direction. The phase angle increases in the direction of the orbital motion and is defined as

$$\tau_i = \frac{(t_i - t_0)}{p}2\pi,$$

where $(t_i - t_0)$ is the time past the x-z plane crossing (at $\tau = 0$) and $p$ is the halo orbit period. Figure 6 shows some values of $\tau$ on a halo orbit which has been projected onto the x-y plane. The phase angles are not evenly spaced because the velocity of the spacecraft varies along the halo orbit.

![Figure 6 Positions of various values of the phase angle, $\tau$, on the halo orbit LL1D projected onto the x-y plane.](image)

With Halo$_2$ constellations and various initial phase angles, the Kalman filter was used to investigate how long it took to converge on a good solution. Figure 7 shows a graph of how the covariance decreased over time for one example.

After looking at these Kalman filter results for several different halo orbits, it was found that it took about 1.5 orbit periods for the solution to converge, regardless of the halo orbit period. This means that, operationally, the halo orbits with a shorter period will lead to quicker and more accurate navigation. In picking the fit span, it was decided to use 1.5 Orbit 1 periods for all tests involving Halo$_2$ and Halo-Halo constellations. In Halo-Earth and Halo-Moon constellations, the error in the lunar or terrestrial gravity field precludes very long integration times in actual operation. Even though the Moon was modeled as a point mass in these tests, the integration time was shortened to 0.5 Orbit 1 periods.
Figure 7 Plot of covariance in time showing convergence using the Kalman Filter for a LL1B Halo₂ constellation with $\Delta \tau = 90^\circ$.

Figure 8 Navigation accuracy for two spacecraft on a halo orbit with varying separation in phase angle $\tau$.

With both spacecraft starting at different phase angles in a Halo₂ constellation, the difference in phase angle, $\Delta \tau = \tau_2 - \tau_1$, remains constant. Figure 8 shows how the orbit determination accuracy varies with $\Delta \tau$. 
The main thing to note from Figure 8 is that the orbit determination error goes up as the spacecraft get close to each other at \( \tau_2 \approx 0^\circ \) and \( \tau_2 \approx 360^\circ \). In fact, the entire state vector becomes unobservable as the spacecraft get too close. While the shape of the error in the middle of the plot varies for different halo orbits, the large error at the edges occurs for every halo orbit. This is because the magnitude of the relative motion of the spacecraft decreases as the distance between them decreases. Even Halo-Halo constellations can have large error if the spacecraft remain close to each other throughout the fit span. This shows that Liaison Navigation would not work for spacecraft in close formations, but only for spacecraft with large separations.

The value of the error metric, \( \beta_{ave} \), varies with the difference in phase angle, so it became necessary to find one representative value of \( \beta_{ave} \) for each combination of orbits. This metric will be called \( \beta_{con} \) and is used to rank the navigation accuracy of the different constellation geometries. In order to find a value of \( \beta_{con} \) that would be a good approximation of the true constellation navigation accuracy, the value of \( \beta_{ave} \) was computed for the following values of \( \Delta \tau \) (in degrees):

\[
\begin{bmatrix}
0 & 7.5 & 15 & 30 & 60 & 90 & 120 & 150 & 180 & 210 & 240 & 270 & 300 & 330 & 345 \\
\end{bmatrix}
\]

It was also found that the value of \( \beta_{ave} \) varied slightly when values of \( \tau_1 \) and \( \tau_2 \) themselves were changed while holding \( \Delta \tau \) constant. To account for those variations, the value of \( \beta_{ave} \) was computed three different times with all the values of \( \Delta \tau \) listed above. \( \tau_1 \) was set to 0, 90 and 180 degrees for the three different runs. For one value of \( \Delta \tau \), the lowest \( \beta_{ave} \) of the three is selected. Of those selected values of \( \beta_{ave} \), one is chosen to reflect the overall navigation accuracy of the constellation. If the two spacecraft were in orbits of the same period, the lowest of the selected values of \( \beta_{ave} \) is used to represent the accuracy of that constellation. However, if the spacecraft are in orbits of differing periods, it is impossible to maintain a constant \( \Delta \tau \). Because of that, the navigation accuracy will vary as the \( \Delta \tau \) changes. In that case, the highest \( \beta_{ave} \) is used to represent the error for that constellation. The resulting error in this approximation was judged to be sufficiently small that a good ranking of constellations could be found using \( \beta_{con} \).

**CONSTELLATION GEOMETRY COMPARISONS**

Initial tests of Halo_2 constellations at LL_1 showed that the navigation error was large for the LL1A and LL1AA orbits, which were the closest to the x-y plane. The x-y plane is the only plane in the CRTBP about which the vector field of accelerations and its time derivative are symmetric. Because of this symmetry, libration orbits become more planar as they approach the x-y plane. When both spacecraft are in the same plane, no observations have any out-of-plane component, and thus it is impossible to estimate the out of plane position and velocity. To verify this conclusion, several more LL_1 halo orbits were integrated close to the x-y plane and the values of \( \beta_{con} \) were separated into x, y, and z components and plotted in Figure 9. For this plot, observations were computed at varying time intervals so that each Halo_2 constellation would produce the same number of observations in 1.5 halo orbit periods. It appears that the z-component of the position is not well resolved until the value of maximum \( |z| \) is at least 0.04 nondimensional units.

It can also be seen from Figure 9 that the error for LL_1 Halo_2 constellations has local minima around maximum \( |z| \) values of 0.06 and 0.2, with the lower minimum occurring around 0.2. It is still not known why the orbits near 0.2 result in better orbit accuracy using Liaison Navigation, but the results from LL_2 Halo_2 constellations follow a similar pattern. The more stable orbits seem to have better orbit accuracy, although the relationship between stability and orbit accuracy has not been determined.

Values of \( \beta_{con} \) were computed for Halo_2 constellations at LL_1, LL_2, and LL_3. \( \beta_{con} \) was also computed for all Halo-Halo constellations in the LL_1, LL_2 and LL_3 test orbits, as well as Halo-Halo constellations with one spacecraft at LL_1 and the other at LL_2. In addition, \( \beta_{con} \) was computed for all the possible Halo-Moon and Halo-Earth constellations. The results of the simulations showed that the initial position and velocity vectors of both spacecraft were observable for most constellations using Liaison Navigation. Some of the best constellations had values of \( \beta_{con} \) that were less than 1 m, while the worst constellation geometries resulted in an unobservable orbit determination problem. Although not depicted in these results, Liaison Navigation also works for constellations with one spacecraft in Earth orbit and one in lunar orbit. Figures 10 through 16 show \( \beta_{con} \) for all possible constellations. The shading of the boxes gives a visual indicator of \( \beta_{con} \), with lighter shading indicating lower
values. The value of $\beta_{\text{con}}$ is also printed in each box. For example, in Figure 10, the top left box labeled 1228.3 corresponds to the constellation where Orbit 1 is LL1A and Orbit 2 is LL1A. The bottom left box labeled 2.9 corresponds to the constellation with LL1FF for Orbit 1 and LL1A for Orbit 2. Note that the boxes are blank in the top right corners of Figures 10, 11, and 12 due to symmetry in the results.

$\beta_{\text{con}}$ for constellations including spacecraft in halo orbits with low vertical components, such as LL1A or LL1AA, were the worst. The results generally get better as the two halos move farther from the plane. This pattern was repeated for the LL$_2$ and LL$_3$ halo orbits also.

The length of the halo orbit periods generally decreases from A to F for all of the halo families. The best results in Figures 10, 11 and 13 were obtained when Orbit 1 was a long period halo and Orbit 2 was a short period halo. This was because the spacecraft in Orbit 2 did much more than 1.5 orbits in the time it took the spacecraft in Orbit 1 to do its required 1.5 orbits. The more revolutions a spacecraft can perform within the fit span, the more geometrically diverse observations can be obtained and the error is reduced.

The LL$_3$ constellations featured in Figure 12 resulted in orbit determination errors that were around 10 times larger than the LL$_1$ and LL$_2$ constellations. This seems to agree with the plot in the previous paper by Hill, et al. which showed that the strength of the asymmetry of the third-body perturbation was weak at LL$_3$. In addition, the LL$_3$ Halo-Earth constellations in Figure 16 had very poor observability. This is because most of the orbit dynamics at LL$_3$ are due to the Earth’s gravity, not the Moon’s.

For the Halo-Moon constellations in Figures 14 and 15, the ones containing the Lunar orbits La, Laa, or Ld seem to have conspicuously worse results. These are the circular, 100 km altitude Lunar orbits. Because they are so low, more of observations are blocked by the Moon. While most Halo-Moon constellations had about 0-10% of the observations blocked by the Moon, the low-altitude constellations had between 25 and 40% of the observations blocked.

Figure 9 $\beta_{\text{con}}$ separated into $x$, $y$, and $z$ components for LL$_1$ Halo$_2$ constellations.
Figure 10  Plot of $\beta_{con}$ (m) for Halo$_2$ and Halo-Halo constellations at LL$_1$.

<table>
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<tr>
<th></th>
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<th>LL1C</th>
<th>LL1D</th>
<th>LL1E</th>
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Figure 11  Plot of $\beta_{con}$ (m) for Halo$_2$ and Halo-Halo constellations at LL$_2$.

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<tr>
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Figure 12 Plot of $\delta_{con}$ (m) for Halo and Halo-Halo constellations at LL$_3$.

Figure 13 Plot of $\delta_{con}$ (m) for Halo-Halo constellations with one spacecraft at LL$_1$ and one at LL$_2$. 
Figure 14 Plot of $\beta_{\mathrm{con}}$ (m) for Halo-Earth and Halo-Moon constellations at LL$_1$.

Figure 15 Plot of $\beta_{\mathrm{con}}$ (m) for Halo-Earth and Halo-Moon constellations at LL$_2$. 

...
ORBIT DETERMINATION PARAMETER COMPARISONS

Out of the many different constellation geometries, 18 were selected for further study and called “baseline” orbits. Table 2 shows the 18 baseline orbit constellations and the resulting values of $\beta_{\text{con}}$ using the baseline test settings as outlined previously. These baseline orbits were used in different test cases to find how the covariance of the orbit estimates change when certain parameters are varied. For each test case, one or two settings were changed, and the rest of the settings were the same as in the preceding analysis (the baseline settings).

Case 1:

The same time period was used for all constellations: 4 nondimensional time units or 17.4 days. In general, constellations with shorter orbit periods did better. This is because observations from all possible vantage points are obtained sooner. As expected, the Halo-Moon orbits fared much better with this longer fit span. This shows that the Halo-Moon constellations are able to recover orbit information faster than the Halo$_2$ or Halo-Halo constellations. However, due to uncertainties in the lunar gravity field, orbit integration times would probably have to be shorter when one spacecraft is in lunar orbit. Also, the errors in the lunar gravity field were not modeled in these simulations. These would offset some of the advantages for Halo-Moon constellations.

Case 2:

The data rate was changed so that observations were taken less often. A measurement was computed every 0.002 nondimensional time units ($\approx$750 seconds) instead of every 0.001 time units. The resulting error increased consistently for all baseline orbits by an average of 41%.
Table 2 DESCRIPTION OF BASELINE ORBITS.

<table>
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<th>Baseline orbit number</th>
<th>Orbit 1</th>
<th>Orbit 2</th>
<th>Baseline Value of $\beta_{con}$ (m)</th>
<th>Constellation Type</th>
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</tr>
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<td>Lf</td>
<td>6.7</td>
<td>Halo-Moon</td>
</tr>
</tbody>
</table>

Case 3:

The effect of a sinusoidal error in the observations was computed using the consider covariance technique outlined in Chapter 6 of Tapley et al.$^5$ The amplitude and the frequency of the sine wave were the consider parameters, with the nominal amplitude modeled as 0.5 m and the period as the shorter of the two orbit periods. The standard deviations of the error in those parameters was 0.5 m for the amplitude and 1/4 period for the frequency. The phase of the sinusoid was assumed to be zero at $t_0$. This increased the values of $\beta_{con}$ by an average of 600%.

Case 4:

The effect of an error in the force model on the covariance was computed using the consider covariance technique. A constant error in the acceleration for the x, y, and z directions was modeled as three consider parameters for each satellite. The standard deviation for the error in the accelerations was set at $1 \times 10^{-8}$ m/s$^2$. The resulting RSS magnitude of the $1\sigma$, 3D unknown acceleration vector would be about 26% of the magnitude of the acceleration due to solar radiation pressure for a 1000 kg spacecraft with a 10 m$^2$ cross section and reflectivity of 1.5. These errors in acceleration increased the values of $\beta_{con}$ quite dramatically for most orbits. However, as a general rule, the Halo-Moon constellations faired better.

Case 5:

An error in the range measurements was introduced in the form of a range bias error with a standard deviation of 5 m. The range bias was added to the state to be estimated along with the positions and velocities of both spacecraft. The results showed that it is possible to estimate the range bias using only SST. However, the overall error in the position increased. There was an average increase of 53% in the value of $\beta_{con}$.
Case 6:

The range bias was estimated as well as including errors in the force model in the form of consider covariance parameters. The resulting increases in \( \beta_{con} \) varied widely. While some values of \( \beta_{con} \) increased by many thousands of percentage points, constellation 14 had an increase of only 0.3% in the error metric.

Case 7:

Doppler measurements were used instead of range. The noise on the Doppler observations was modeled as Gaussian with a standard deviation of 0.5 mm/s. The orbit estimate errors increased for all of the baseline constellations by an average of about 2900%, confirming that Doppler would indeed be a weaker data type.

Case 8:

Because of the large link distances involved, it is probable that highly directional antennas would need to be used when generating the SST range measurements. Unless the spacecraft have more than one high gain antenna, they would not be able to continuously track each other and communicate with other entities simultaneously. Because of that, observations were taken in two-hour blocks separated by two-hour periods without any measurements to simulate link periods when the spacecraft were communicating with landers, tracking user spacecraft, or communicating with ground controllers. The resulting increase in \( \beta_{con} \) averaged 46% for all the baseline constellations.

Case 9:

This run was meant to simulate as close as possible the orbit accuracy attainable in a real operations environment. One final run was performed using the LL2c and Lc orbits. This was one of the better constellations, although it wasn’t the best. In this run, the interval between observations was reduced to 0.0001 time units (\( \approx 37.5 \) seconds), which required a much larger ephemeris file. Consider covariance analysis for unknown accelerations was performed as before with the range bias included in the estimated state vector. Two-hour communication periods were separated by two-hour periods with no observations. The standard deviation on the range measurement noise was set to 1 m and Doppler measurements were not used. A range of phase angle values was used for satellites 1 and 2 and the worst-case \( \beta_{con} \) was 9.79 m. For that case, the maximum range between the spacecraft was about 86,000 km. The \( \beta_{ave} \) for the satellite in the halo was 17.4 m and 2.4 m for the satellite in lunar orbit. The number of observations for 0.5 LL2c orbit periods (\( \approx 3.6 \) days) was 3805, with 384 observations blocked by the Moon. Due to uncertainties in the lunar gravity field, a 3.6-day lunar orbit propagation might not be possible. However, the lunar orbits of Lunar Prospector were successfully propagated for 2.3-day fit spans, and Lunar Prospector was in a lower orbit than Lc.

MONTE CARLO ANALYSIS OF THE COVARIANCE

To verify that the covariance matrices used were correct, a Monte Carlo analysis was conducted using the LL2c-Lc Halo-Moon constellation. The spacecraft in LL2c was placed at \( \tau = 0^\circ \), and the spacecraft in Lc was placed at \( \nu = 90^\circ \) (true anomaly). Observations were processed every \( \approx 375 \) seconds with 2 hour blackout periods. A range bias was added to the state vector and the fit span was 0.5 LL2c orbit periods long. The satellite states were perturbed by normally distributed random numbers with a 1\( \sigma \) of \( 1 \times 10^{-6} \) (about 384 m for position and 1 mm/s for velocity). The range bias was perturbed with a normally distributed random number with a 1\( \sigma \) of 5 m. After converging on a solution, the estimated state was compared to the true state. This was done 1000 times. If the covariance matrix is realistic, the orbit error will be inside the 3D, 2\( \sigma \) error ellipsoid for 73.9% of the runs. To find if the error is inside the 2\( \sigma \) error ellipsoid, the size of the error ellipsoid that intersects the orbit error vector is computed for each satellite. The size of the ellipsoid, \( L_i \), has units of standard deviations (\( \sigma \)) and is computed as shown in Tapley, et al. section 4.16.

\[
L_i^2 = \mathbf{x}_i^T P_i^{-1} \mathbf{x}_i
\]
\( \mathbf{x}_i \) is the error vector for spacecraft \( i \), or the true state subtracted from the state estimate. \( P_i \) is the \( 3 \times 3 \) position covariance matrix for spacecraft \( i \). For the LL2c-Lc constellation, 74\% of the values of \( L_1 \) and 73\% of the values of \( L_2 \) were below \( 2\sigma \). For the range bias covariance to be accurate, the range bias error should be less than \( 2\sigma \) for 95.4\% of the runs. The range bias error was within \( 2\sigma \) for 98\% of the range bias errors.

The LL1c-LL1d Halo-Halo constellation was also tested in the same way using 1000 runs. The spacecraft in LL1c was placed at \( \tau = 0^\circ \), and the spacecraft in LL1d was placed at \( \tau = 90^\circ \). Observations were processed every \( \approx 375 \) seconds with 2 hour blackout periods. A range bias was added to the state vector and the fit span was 1.5 LL1c orbit periods long. The satellite states were perturbed by normally distributed random numbers with a \( 1\sigma \) of \( 1 \times 10^{-7} \) (about 38 m for position and 0.1 mm/s for velocity). The range bias was perturbed with a normally distributed random number with a \( 1\sigma \) of 5 m. For this constellation, 72\% of the values of \( L_1 \), 71\% of the values of \( L_2 \), and 97\% of the range bias errors were below \( 2\sigma \). For the constellation LL2c-Lc constellation, the covariance appears to be accurate, but optimistic by a few percentage points for the constellation LL1c-LL1d. The optimism in the LL1c-LL1d covariance could be due to correlation between the two satellite states.

**CONCLUSION**

Based on the simulations in this study, Liaison Navigation in lunar halo orbits not only appears possible, but also has the potential to be highly accurate. Remember that the \( \beta_{ave} \) and \( \beta_{con} \) metrics are based on the \( 3\sigma \) error ellipsoid. Based on these simulations, it seems plausible that an actual application of Liaison Navigation in lunar halo orbits would be able to achieve a \( 1\sigma \) orbit accuracy around 10 m. If so, the improved orbit accuracy for spacecraft in libration orbits provided by Liaison Navigation would lead to a decrease in the \( \Delta V \) needed for stationkeeping, since more accurate knowledge of the orbit leads to smaller stationkeeping maneuvers.

When selecting halo constellation geometries for use in Liaison Navigation, orbit determination accuracy is improved if the following conditions are met.

- Spacecraft should have relatively large separation distances.
- All spacecraft should not remain in the same plane for their entire trajectories.
- Libration orbits with shorter periods lead to quicker convergence.

These simulations did not include perturbations to the CRTBP. However, well-known perturbations could make the Liaison orbit determination problem more observable if they add asymmetry and the perturbations could be accurately modeled. The perturbations could lead to worse orbit accuracies if there is indeed a relationship between stability and orbit accuracy and the perturbations make the orbits more unstable.

Using Liaison Navigation, a constellation of spacecraft in lunar libration orbits could be used as a communication relay for lunar orbiters and landers as well as a tracking network to provide lunar surface positioning and orbit determination. This would be a significant asset in carrying out the President’s vision for exploration of the Moon. This would also relieve some of the burden on the DSN and could possibly represent better orbit accuracy than that attainable through the DSN. One advantage of autonomous orbit determination is that nearly continuous coverage is possible. While continuous coverage using Earth-based tracking would probably be more accurate than Liaison Navigation in general, that might not be achievable considering the current high demand for DSN assets.

While only lunar halo orbits were simulated in this study, Liaison Navigation should also work for quasi-periodic or Lissajous orbits in the Earth-Moon system as well as libration orbits in any other three-body system, such as the Sun-Earth system.
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REFERENCES


