CORRECTION ALGORITHMS FOR GPS CARRIER PHASE MULTIPATH
UTILIZING THE SIGNAL-TO-NOISE RATIO AND SPATIAL CORRELATION

by

ANGELA KAYE REICHERT

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written by

Angela Kaye Reichert

has been approved for the

Department of Aerospace Engineering Sciences

by

Penina Axelrad

Kristine Larson

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CORRECTION ALGORITHMS FOR GPS CARRIER PHASE MULTIPATH REDUCTION UTILIZING THE SIGNAL-TO-NOISE RATIO AND SPATIAL CORRELATION

Thesis directed by Associate Professor Penina Axelrad

GPS Multipath is caused by reflections of the signal from nearby surfaces and produces significant errors in both the GPS pseudorange and the carrier phase. In this research, algorithms are developed to reduce multipath found in the GPS carrier phase measurement. Multipath errors are estimated using a spatial SNR-based mitigation technique, as well as a spatial phase-based method. Once the errors have been identified, corrections are applied to both simulated and real data in order to assess the phase improvement. Unfortunately, the method is not effective in identifying a reflector capable of removing multipath in the flight data from CRISTA-SPAS. However, the results using ground data display improvements in the RMS of the residuals of up to 22 percent, and indicate further potential for multipath reduction capabilities using the reflector identification algorithm.
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CHAPTER 1

INTRODUCTION

It has been widely noted that the applications of the Global Positioning System (GPS) have grown tremendously since the initial operation of the system in the early 1990's. Along with this explosion of applications has come an ever increasing demand for improved accuracy. Modern receiver systems routinely produce phase-based solutions for vehicle attitude, and both static and kinematic relative positioning. As advances begin to push the accuracy boundaries of phase-based processing, error sources once thought to be negligible begin to drive performance. Multipath is one such error source. Multipath is caused by reflections of the signal from nearby surfaces and produces significant errors in both the GPS pseudorange and the carrier phase observable.

1.1 Objectives and Motivation for Research

The purpose of this research is to develop algorithms to reduce multipath found in the GPS carrier phase measurement. Multipath errors are estimated using a spatial SNR-based mitigation technique, as well as a spatial phase-based method. Once the errors have been identified, corrections are applied to both simulated and real data in order to assess improvement in phase.

This research is motivated by the fact that carrier phase multipath continues to be a significant source of error for many phase-based applications. The dominant error source found in GPS attitude determination is phase multipath [Cohen, 1995]. Applications which
use a reference and a remote antenna, such as real-time kinematic surveys and time transfer applications, are susceptible to multipath because of the highly localized characteristics of multipath, thus making it unlikely that the same error trends exist between the reference station and the remote antenna [Braasch, 1995].

1.2 Global Positioning System

Developed by the Department of Defense (DoD), the Global Positioning System (GPS) is comprised of a constellation of 24 satellites orbiting at an altitude of approximately 20,000 km and a network of monitoring sites to support them. The primary purpose of GPS is to provide continuous position, velocity, and time information to a user on or near the Earth's surface. Since GPS is a passive system, it is capable of simultaneously supplying any number of users with navigation information. A comprehensive overview of GPS operations and applications is presented in Parkinson et al. [1996].

The current GPS transmissions include two L-Band carrier signals at 1575.42 MHz (L₁) and 1227.6 MHz (L₂), bi-phase modulated by one or more pseudo random noise (PRN) codes. The PRN codes permit multiple satellites to operate at the same frequency and enable receivers to make direct measurements of the signal transit time from the satellite to receiver. Two types of codes are used, the Clear Acquisition (C/A) code generated at a chipping rate of 1.023 MHz and used primarily by civilians, and the Precise P (Y) code generated at 10.23 MHz and used for military applications. A low frequency "Navigation Data" stream describing the satellite orbits and other system parameters is modulo 2 added to the codes prior to carrier modulation.

The L₁ carrier signal transmitted from the GPS satellites is right-hand circular polarized (RCP) and is received using an RCP antenna. An circularly polarized signal is used for two reasons. The first is to avoid the effects of Faraday rotation, which is the rotation of a
linear polarized signal as it passes through the Earth’s ionosphere [Klobuchar, 1994]. The second reason to use an circularly polarized carrier is to reduce multipath. After an RCP signal reflects it becomes left-hand circularly polarized (LCP). When a typical RCP antenna receives an LCP multipath signal, the multipath amplitude is greatly reduced, and in some cases it is completely rejected by the antenna.

The GPS receiver unit typically includes an antenna and preamplifier, down conversion section, signal processor, and navigation processor. The antenna couples the incident electromagnetic energy into current and voltage signals which are then amplified and filtered by the low noise amplifier (LNA) in the preamplifier. The down conversion section shifts the signal center frequency from the L-band radio frequency to an intermediate or baseband frequency where conversion from analog to digital representation is performed. The signal processor handles all signal tracking functions required to acquire the signals and make measurements of range, Doppler, and extract the encoded navigation data. The navigation processor performs the application-specific tasks such as forming position and velocity solutions. The two components of the receiver system which have a significant impact on the contribution of multipath to the measurements are the antenna and the signal processor. These functions are discussed in detail in subsequent chapters.

1.3 Trimble Advanced Navigation Sensor Vector

One type of receiver used for GPS attitude determination is the Trimble Advanced Navigation Sensor (TANS) Vector receiver. It was developed by C. Cohen at Stanford University in conjunction with Trimble Navigation Limited [Cohen, 1992], [Trimble 1993]. Data sets collected from a TANS Vector receiver are presented in this dissertation from both ground experiments and the CRISTA-SPAS spacecraft.
The TANS Vector has six channels and is designed to compute phase measurements from four antennas – one master and three slaves. The four antennas share a common local oscillator, which eliminates the need to estimate a separate clock bias for each antenna. The phase measurement reported for the master antenna is similar to the phase measurements from conventional positioning receivers. The phase from the slave antennas, however, is referenced to the phase from the master antenna. It is reported as the difference in phase from the master, and is referred to as the differential phase.

In addition to the differential phase measurement, the receiver also reports the signal-to-noise ratio (SNR) for each of the individual antennas. For the TANS Vector, the SNR values are reported in Trimble Amplitude Units (AMU). The AMU is related to the more conventional carrier-to-noise ratio, $C/N_0$, as shown below [Comp, 1996]:

$$C / N_0 \text{ (dB - Hz)} = 10 \log_{10} \left( BW \cdot A^2 \right).$$

The variable $A$ is the SNR as specified in Trimble Amplitude Units (AMU) and $BW$ is 1 kHz for the TANS Vector. Factors which affect the SNR are satellite signal transmit power, atmospheric losses, receive antenna gain pattern, correlator parameters, and multipath.

1.4 Previous Work

GPS attitude determination is a widely studied topic because of the advantages GPS represents as a potential low-cost attitude sensor for spacecraft. Not until recently did commercial receivers become available for spacecraft attitude determination. One such receiver, the Trimble TANS Vector [Trimble, 1993; Cohen, 1992], has been successfully flown on board a number of Earth-orbiting spacecraft, such as the U.S. Air Force RADCAL [Ward and Axelrad, 1995], CRISTA-SPAS [Brock et al., 1995; Ward, 1996], REXII [Lightsey et al., 1994], and on the Space Shuttle GANE [Carpenter and Hain, 1997; Axelrad and Highsmith, 1998].
Current results show that when using GPS alone for attitude determination, the accuracy is often limited by multipath. This is evidenced in the wandering structure of the residual error [Ward, 1996]. In general, multipath contributes approximately 5 millimeters of correlated error to the GPS carrier phase, and thus is the largest source of error in GPS attitude determination [Cohen, 1995]. The correlated nature of multipath makes it particularly difficult to correct using additional systems such as gyros, where the multipath may alias into the gyro bias. In order to try to compensate for multipath errors, many methods have been suggested for multipath rejection, including modeling, estimation, and mitigation. Some of these methods are described below.

One approach is to correct for multipath within the receiver. A few of the proposed internal receiver techniques include narrow correlators [Van Dierendonck et al., 1992], strobe and edge correlators [Garin et al., 1996], and the multipath estimating delay lock loop (MEDLL) [Townsend et al., 1995]. These methods can potentially reduce the amount of multipath in the observables at the expense of increased receiver complexity.

Another approach is to model the multipath for a specific antenna and environment using the geometrical theory of diffraction (GTD) [Hajji, 1990 and Lippencott et al., 1993]. This approach requires a knowledge of the antenna gain pattern and the geometry and reflectivity of objects surrounding the antenna. Based on this information, a ray-tracing algorithm determines a model of the resulting phase and amplitude of the multipath error. Phase errors produced by this simulation have been confirmed in ground experiments [Gomez et al., 1995 and Irish et al., 1998].

A procedure based upon the spatial correlation characteristics of multipath was first proposed by Cohen and Parkinson [1991]. Experimental data collected on a fixed ground structure over the course of several days is used to create a map of multipath as a function of the direction of the incoming direct GPS signal. Based on the known attitude and baseline locations of the ground platform, carrier phase residuals are computed and prefiltered to
remove receiver noise. Then, an eighth order spherical harmonic model is fit to the residuals as a function of azimuth and elevation of the incoming signals. Using this approach, Cohen and Parkinson [1991] showed reduction of the residuals from 5.2 to 3.2 millimeters.

Another method for post-processing the GPS carrier phase was implemented by Comp [1996] and Comp and Axelrad [1997]. Instead of modeling multipath in the residual phase, this approach utilizes the SNR in order to estimate multipath. One advantage of using the SNR over previous method is that the SNR is much less sensitive to attitude errors, while the residual differential phase is directly dependent on any errors in the attitude. This SNR-based method analyzes the data over individual satellite passes as a function of time, identifying dominant frequency components in the signal due to multipath. Although this approach showed significant reduction in phase multipath, it is not entirely effective for real-time applications because of the long convergence time required.

Ray et al. [1998] developed a method to reduce multipath by using a system of multiple, closely spaced antennas in a static multipath environment. With the location of the antennas precisely known, it is possible to isolate the direct signal phase and discard the portion of the signal corrupted by multipath. A drawback of this method is that it relies on using the phase measurement, which is highly sensitive to errors in antenna location and attitude. Also, the method relies on using a number of closely spaced antennas. Although it may be possible to use many antennas for some ground stations, the use of multiple antennas would be too costly to use in a spacecraft environment for attitude applications.

1.5 Research Contributions

This research further develops algorithms for correcting GPS carrier phase multipath using post-processing techniques. The research contributions presented in this dissertation can be divided into five categories, covering reflection theory characterization and
development, baseline and line bias calibration, multipath reduction using the spatial
dependence of the phase measurement, multipath reduction using the temporal dependence of
the SNR measurement, and multipath reduction using the spatial dependence of the SNR
measurement.

1.5.1 Reflection Theory Development

The first contribution is the characterization of the behavior of a circularly polarized
signal upon reflection for a perfect conductor. While the behavior of polarized signals at
oblique incidence is well known, the phase of a signal received by a circularly polarized
antenna after reflection is not as thoroughly documented. Because the signal phase is of
lesser importance than signal strength in most communication applications, most discussions
do not focus on the behavior of the signal phase, which is of primary importance in GPS
applications. Using the theory behind the reflection of polarized signals, expressions are
derived to represent the change in phase of the circularly polarized signal after a reflection
occurs, as received by a GPS antenna. The expressions for the phase change are verified by
data from the Numerical Electromagnetic Code-Basic Scattering Code, simulation software.

1.5.2 Baseline and Line Bias Calibration

In many applications, the baseline and line bias values are not precisely known. Accurate
knowledge of these values is important in order to effectively utilize the phase
measurement for applications such as attitude determination or time transfer. An example of
a case where the baseline is not well known is for spacecraft attitude, where the surveyed
antenna positions may differ from the true positions due to either a flexure of the spacecraft
upon launch or changes in the phase center of the antenna once the satellite becomes fully
operational on-orbit. If a sufficiently accurate attitude solution exists from some source
external to the GPS-reported attitude, the baselines and line biases may be calibrated using a
batch post-processing scheme. In this research, an algorithm to perform this calibration is developed and is implemented using data from a ground test and data from the CRISTA-SPAS spacecraft.

1.5.3 Multipath Reduction Using Spatial Dependence of the Phase Measurement

Utilizing the spatial dependence of carrier phase multipath, residual phase sky maps are computed for post-processed correction of multipath. The approach is based upon static algorithms first presented by Cohen and Parkinson [1991] and Cohen [1992]. In this dissertation I present phase maps of a moving CRISTA-SPAS spacecraft, constructed using the spherical harmonic representation as presented by these authors. Further extending their work, maps are constructed using a two-dimensional polynomial fit and a grid fit to the CRISTA-SPAS residual phase data. For each of the methods, the residual phase improvements are noted as well as the corresponding improvements to the attitude solution as a result of the phase corrections.

1.5.4 Multipath Reduction Using Temporal Dependence of the SNR Measurement

The work from Comp [1996], in which multipath is identified by estimating the spectral parameters of multipath from the SNR, is extended in this thesis. The algorithm is applied to all data sets for the CRISTA-SPAS spacecraft in which a sufficient number of data points are available for convergence of the estimator. Corrections are computed for the residual phase data based on the multipath spectral parameters. Using the updated differential phase measurements, improved attitude solutions are computed using the method developed by Ward [1996].
1.5.5 Multipath Reduction Using Spatial Dependence of the SNR Measurement

Finally, a process which utilizes the GPS SNR is developed in order to identify an effective reflector causing multipath. It is termed an effective reflector because the reflector identified can account for a number of effects found in the signal, including multipath due to multiple reflectors and time-varying changes in reflectivity. A physical location is used simply as a means to utilize the spatial correlation of the multipath. The reflector identification process is implemented for simulated data, ground test data, and data from the CRISTA-SPAS spacecraft.

1.6 Dissertation Overview

The remainder of this dissertation is organized as follows. Chapter 2 is comprised of the electromagnetic theory needed to describe the electric fields and the antenna characteristics, and Chapter 3 includes an in-depth discussion of multipath as viewed in the signal-to-noise ratio, the pseudorange, and the carrier phase measurements. In Chapter 4 I present the algorithms used in this research, including a batch algorithm for baseline and line bias calibration, the algorithmic methods for residual phase sky map production, the temporal SNR-based multipath mitigation algorithm, and the reflector identification algorithm. Chapters 5, 6, and 7 contain results for simulated data, ground data, and flight data from the CRISTA-SPAS spacecraft, respectively. Finally, the conclusions of the research and proposed future work are presented in Chapter 8.
CHAPTER 2

ELECTROMAGNETIC THEORY

2.1 Introduction

This chapter provides a background discussion of electromagnetic theory as it pertains to GPS circularly polarized signals. The first section provides an overview of the field regions surrounding an antenna and the representation of the fields in a spherical coordinate frame. A discussion of circular polarization and its relationship to the spherical coordinate frame follows the field descriptions. Finally, GPS antenna patterns are discussed and a sample pattern for the Trimble patch antenna, as used in this dissertation, is presented.

2.2 Electromagnetic Fields

The space surrounding an antenna can be divided into two regions – the near field and the far field [Kraus, 1950]. In the far field, the majority of the electric field is transverse to the direction of travel and the shape of the field is independent of the radial distance from the antenna, while in the near field, the radial component of the electric field is significant. Although the boundary between the near and far field is not rigidly defined, as a general rule the transition from the near to the far field occurs at a radius, $R$, of

$$R = \frac{L^2}{\lambda}.$$  

(2.1)
where $L$ is the maximum dimension of the antenna, and $\lambda$ is the wavelength. However, in order for this equation to be valid, $L$ must be large compared to the wavelength [Balanis, 1997]. Because the maximum dimension for a GPS antenna is typically 10 centimeters and the wavelength is 19.03 centimeters for the L1 frequency, the far field expression above is not valid. However, for the purposes of this research, it will be assumed that the GPS antenna far field exists beyond approximately three to five wavelengths and that all fields analyzed are in the far field.

For far field electromagnetic signals, a spherical coordinate frame is a common way to represent the signal in the far field. A representation of the spherical coordinates of the electric field, $E$, is shown below in Figure 2.1. In order to follow typical electromagnetic theory conventions, the amplitude and phase components of the electric field are represented in terms of complex-valued phasor quantities. In the far field, the radial component, $E_r$, reduces to zero, due to the assumption that all components of the electric field are negligible in the direction of propagation. Therefore, in order to completely describe a far field signal, only the $E_\theta$ and $E_\phi$ components must be known.
2.3 Circular Polarization

The polarization of a wave describes its time-varying amplitude and direction. For a linearly polarized wave, the vector describing the characteristics of the wave is always directed along a line. A special case is circular polarization, where two orthogonal, linearly polarized signals are combined in such a way that the vector describing the wave traces out a circle. A diagram of a circularly polarized signal is shown below in Figure 2.2. This combination of the linear signals can be generated using two crossed dipoles excited 90 degrees out of phase from one another [Stutzman, 1993].

Figure 2.1: Electric field representation in the spherical coordinate frame. The three components of the electric field vector are represented as $E_\theta$, $E_\phi$, and $E_r$. 

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Figure 2.2: Diagram of an RCP signal. The electric field vectors are represented for a far-field CP signal, in which the electric field vectors are always perpendicular to the direction of propagation.

The unit vector of a linearly polarized signal directed in the $y$-direction, as shown in Figure 2.1, is represented in the spherical coordinate frame by the equation [Stutzman, 1993],

$$\hat{e}_{LPy} = \sin \phi \hat{\phi} + \cos \phi \hat{\phi}.$$  \hspace{1cm} (2.2)

Similarly, the unit vector for a linearly polarized signal directed in the $x$-direction is represented by

$$\hat{e}_{LPx} = \cos \phi \hat{\theta} - \sin \phi \hat{\phi}.$$  \hspace{1cm} (2.3)

Note that the unit vectors for the linearly polarized signals are orthogonal to one another. Because an electric field can be described completely by the combination of two orthogonal states, the field radiated from an antenna can be described by the two components in these unit vector equations. Since one direction nominally defines the desired polarization, the equations above describe the copolarized and cross polarized unit vector directions. These cop- and cross polarized equations are based on Ludwig's third definition for cross polarization, which pertains directly to antenna measurements [Ludwig, 1973].

Because the GPS antenna is designed to be right-hand circularly polarized, a common way to define the characteristics of GPS antennas is to specify the right-hand (RCP) and left-hand circularly polarized (LCP) electric field components. Combining the equations above for the two dipoles produces the representation of a circularly polarized signal in the spherical
coordinate frame. The complex unit vectors for the RCP and LCP directions in terms of the spherical coordinate complex unit vectors are shown in equations (2.4) and (2.5) [Stutzman, 1993].

\[ \hat{e}_{RCP} = \frac{1}{\sqrt{2}} \left[ (\sin \phi + j \cos \phi) \hat{\varphi} + (\cos \phi - j \sin \phi) \hat{\varphi} \right] \]

(2.4)

\[ \hat{e}_{LCP} = \frac{1}{\sqrt{2}} \left[ (-\sin \phi - j \cos \phi) \hat{\varphi} + (\cos \phi + j \sin \phi) \hat{\varphi} \right] \]

(2.5)

In these complex unit vector equations, \( \phi \) is the azimuth of the incoming signal in the spherical coordinate frame. The equations have been modified from the ones in Stutzman [1993] to account for the opposite sign of the azimuth for the RCP and LCP definitions.

A matrix to transform the electric fields from the circularly polarized frame into the spherical coordinate frame is derived by manipulating the complex unit vector equations shown in (2.4) and (2.5). The equations to transform from the circularly polarized frame to the spherical frame and vice versa using the transformation matrix, are shown in equations (2.6) and (2.7), respectively.

\[ \begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \sin \phi + j \cos \phi & -\sin \phi - j \cos \phi \\ \cos \phi - j \sin \phi & \cos \phi - j \sin \phi \end{bmatrix} \begin{bmatrix} E_{RCP} \\ E_{LCP} \end{bmatrix} \]

(2.6)

\[ \begin{bmatrix} E_{RCP} \\ E_{LCP} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \sin \phi - j \cos \phi & \cos \phi + j \sin \phi \\ -\sin \phi + j \cos \phi & \cos \phi + j \sin \phi \end{bmatrix} \begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} \]

(2.7)

Note that in these equations, the amplitude and the phase of the signals are represented using complex notation, where the magnitude of the electric field vector is the amplitude of the signal as defined by

\[ \text{amplitude} = \sqrt{(\text{Re}(E))^2 + (\text{Im}(E))^2} \]

(2.8)

and the phase of the signal is defined by

\[ \text{phase} = \arctan \left( \frac{\text{Im}(E)}{\text{Re}(E)} \right) \]

(2.9)
where the specific quadrant for the phase is determined by the signs of the numerator and denominator terms.

2.4 Antenna Patterns

An antenna pattern is a representation of the far field radiation properties of an antenna and is usually specified in a spherical coordinate system [Stutzman and Thiele, 1981]. The antenna pattern is composed of the amplitude and phase of the electric field variations as a function of the azimuth and elevation of the incoming signal. Because GPS signals are circularly polarized, the patterns are usually presented for both the LCP and RCP components of the signal. Ideally the antennas are designed in order to minimize the amplitude of the LCP pattern so that most of the LCP signals are rejected.

The GPS antenna amplitude pattern, also referred to as the gain pattern, is shaped in such a way that it drops off dramatically with decreasing elevation angles, while providing a broad enough pattern to receive signals from at least four GPS satellites. The antenna is designed in this way so that signals being received at very low and negative elevation angles are significantly attenuated. The rejection of signals from lower elevations aids in reducing the effects of multipath because these signals arriving at low elevation angles are more likely to have reflected off of a nearby object, such as a mounting surface or the ground, before entering the antenna.

The antenna phase pattern is a measure of the phase center variation as a function of azimuth and elevation of the signal line of sight. The phase center is defined as the point in space for which there is the smallest variation in received phase for all incoming signals [Schupler et al. 1994]. The phase center rarely coincides with a point on the antenna and is usually located at some point a centimeter or more above the antenna. An in-depth discussion
of GPS gain and phase patterns, along with a survey of the characteristics of a number commonly used GPS antennas can be found in Schupler et al. [1994].

For the data presented in this dissertation, the antenna pattern used is one specified for a Trimble patch antenna. The amplitude pattern for this antenna was provided by R. Allshouse and is shown below in Figure 2.3 [Comp, 1996]. While the effects of phase center variation are often included in geodetic applications, in this research the phase pattern for the Trimble patch antenna is assumed to be uniformly zero, corresponding to a negligible phase center variation as a function of the line of sight. This assumption is valid for the applications in this research because I only work with small separations of identical antennas. In this case the antenna phase variation present in the direct signal phase is removed when the differential phase measurement is formed between the two antennas. Furthermore, the effects due to phase center variation existing in the multipath signal are compensated for by estimating a reflector location which accounts for the phase center effects in the multipath relative phase.
Figure 2.3: Gain pattern for the Trimble patch antenna, provided by R. Allshouse.
CHAPTER 3

MULTIPATH BACKGROUND

3.1 Introduction

This chapter begins with a general discussion of multipath, and follows with an overview of the behavior of pseudorange and carrier phase multipath. Expressions are derived for the composite signal-to-noise ratio as a function of the multipath relative phase and the multipath and direct signal amplitudes. Finally, the chapter concludes with an in-depth discussion of the multipath relative phase by investigating the signal characteristics due to both the physical path delay and the phase change of a polarized signal upon reflection.

3.2 Overview of GPS Multipath

Multipath occurs when an electromagnetic signal arrives at an antenna, not along a direct path, but via one of the indirect paths the signal takes as it reflects off an object near an antenna. It may either be diffuse or specular in nature. Specular multipath is problematic because it produces systematic, time correlated errors that are not easily treated in a processing filter. In contrast to the correlated nature of specular multipath, diffuse multipath takes on an unbiased, random appearance, and is more easily removed through filtering than specular multipath. Therefore, only specular multipath will be considered in this research. For simplicity, the theoretical development of the characteristics of multipath presented here is based on a perfectly conducting surface.
Multipath continues to be a significant problem for many GPS applications using either the carrier phase or the pseudorange measurement. One of the most significant problems multipath poses is that it is not correlated between different sites, as is the case with other GPS errors such as atmospheric errors due to the ionosphere and troposphere. Thus, multipath cannot be corrected in a mobile station by sending out corrections from a reference station.

A number of methods for the rejection of multipath already exist. However these approaches have limitations. Under ideal situations, the easiest method for correction of multipath errors is to place GPS antennas in such a way as to avoid multipath. However, quite often this is not possible due to siting constraints. More complex receiver architecture can be utilized which reduces multipath by using narrow correlators [Van Dierendonck et al., 1992] or other techniques such as the Multipath Estimating Delay Lock Loop (MEDLL) [Townsend et al., 1995]. The use of more sophisticated receiver design can often be too costly. A third technique which is frequently used for multipath rejection is a choke ring antenna. This type of antenna is rather large and can be impractical for certain types of applications, such as spacecraft attitude determination, where space for mounting GPS antennas is very limited in most cases.

Another means to reduce the effects of multipath is to allow it to exist in the signal and then to remove it by post-processing the data. This method was investigated by Comp [1996] using a temporal SNR-based multipath mitigation algorithm and by Ray et al. [1998] using multiple antennas arranged in an array in order to determine multipath spatial characteristics. While Comp's method can be applied to any data set containing multipath, that of Ray et al. requires an array of five closely spaced antennas. Antenna configurations of this sort are possible in ground applications, but are unlikely to be used in space applications.

A method is developed in this thesis to post-process data from a single antenna using the SNR and the characteristics of spatially correlated multipath in order to identify a
reflector causing multipath and to compute corrections in the carrier phase measurement. The theoretical background for this type of multipath modeling is developed in the remainder of this chapter.

3.3 Multipath in the GPS Pseudorange, Carrier Phase and SNR

The GPS signal, comprised of the code and the carrier, is received as a spread spectrum signal. After down-converting the received signal from the $L_i$ to an IF frequency, the received signal is correlated with a locally generated code inside the delay lock loop (DLL). In the DLL, the local code is shifted in time until the correlation peak is at a maximum. The time shift is used to compute the signal transit time, thus producing the pseudorange measurement. The recovered carrier signal is output from the DLL. The carrier signal is then fed into the phase lock loop (PLL) where it is matched with a locally generated carrier using a numerically controlled oscillator (NCO). Once the receiver has a lock between the sinusoids, the change in phase of the signal over time is recorded.

Multipath occurs in the pseudorange because the true signal correlation peak is masked by the addition of another correlation peak due to the multipath signal [Parkinson, 1994]. Two illustrations of this phenomenon are shown in Figure 3.1 and Figure 3.2. In Figure 3.1, the diagram on the left represents the correlation peak for the direct signal in bold and the multipath signal with a dotted line. The horizontal axis indicates the chip spacing of $T$, where zero represents the true pseudorange time delay. The multipath correlation peak lags the direct signal correlation peak by $\delta_m$, which corresponds to the time delay in the multipath signal. Multipath affects the correlation process by combining with the direct signal to create a distorted correlation peak, thus masking the identification of the true time delay, as illustrated in the diagram on the right. The correlator identifies the peak by locating two points where the function returns the same value and inferring that the peak lies directly
between the two values. The correlator process is illustrated on the second diagram for the distorted function value. In this example where the multipath and direct signals are in phase, the correlator returns an erroneous peak value of $\delta t$.

Figure 3.2 illustrates a similar process to the one shown in Figure 3.1, but here the multipath is 180 degrees out of phase from the direct. The multipath also lags the direct signal by $\delta m$, but instead of causing an erroneous correlation peak value of $\delta t$, as in the case of the multipath and direct adding in phase with one another, the peak is off by $-\delta t$. This example illustrates how pseudorange multipath can cause negative as well as positive ranging errors in the signal, even though the multipath always delays the direct signal.

Not illustrated here is the case in which the multipath and direct signal are 90 degrees out of phase. When the signals are 90 degrees out of phase, the multipath does not cause a distortion in the correlation peak, and thus does not cause multipath in the pseudorange.

Figure 3.1: Distortion of the code correlation peak with multipath in phase with the direct signal. The diagram on the left displays the direct and multipath correlation peaks individually, and the diagram on the right illustrates the combination of the two peaks. The resulting pseudorange error is positive.
Figure 3.2: Distortion of the code correlation peak with multipath 180 degrees out of phase from the direct. The diagram on the left displays the direct and multipath correlation peaks individually, and the diagram on the right illustrates the combination of the two peaks. The resulting pseudorange error is negative.

Carrier phase multipath occurs because the phase lock loop in the receiver locks onto the composite signal, composed of the combination of the desired direct signal and the reflected signal. An illustration of the composite signal is depicted in the phasor diagram shown in Figure 3.3. The amplitude values, denoted by $A$, correspond to SNR measurements. The angular values, denoted by $\phi$, correspond to the phase of the signal.
Figure 3.3: GPS signal phasor diagram. The three phasors represent the direct, multipath, and composite signals, where the amplitudes are $A_o A_o$, $A_m$, and $A_c$, respectively. The phase of the direct signal is $\phi_d$ and the phase of the composite is $\phi_c$. The multipath relative phase is $\psi$ and the phase error due to multipath is $\delta \phi$.

In Figure 3.3, $A_c$ is the composite SNR measurement and $\phi_c$ is the composite carrier phase. These are the quantities that the receiver measures. In addition to the composite signal phasor, two others are represented in this figure – the direct signal phasor and the multipath phasor. The direct signal SNR amplitude is represented by $A_o A_o$, where $A_o$ is the normalized antenna gain and $A_o$ is the constant part of the direct signal amplitude. The multipath amplitude is $A_m$ and $\psi$ is the phase of the multipath relative to the direct signal. Using the law of cosines, the equation for the composite SNR in terms of the direct and multipath signals is shown in equation (3.1).

$$A_c^2 = (A_o A_o)^2 + A_m^2 + 2A_o A_o A_m \cos \psi$$  \hspace{1cm} (3.1)
The composite carrier phase measurement, $\phi_c$, can also be described as a function of the multipath amplitude and phase. The phase error due to multipath is the residual phase, $\delta \phi$. An expression for the residual phase is shown in equation (3.2).

$$\tan(\delta \phi) = \frac{A_m \sin \psi}{A_o A_o + A_m \cos \psi}$$  \hspace{1cm} (3.2)

From the phasor diagram, note that as the multipath relative phase, $\psi$, increases or decreases, oscillations occur in the SNR and the phase error, $\delta \phi$, that are approximately ninety degrees out of phase from one another. Thus, the multipath in the SNR is out of phase with the multipath in the carrier. Additionally, the pseudorange multipath oscillates in phase with the SNR measurement. The behavior of the pseudorange multipath follows from the discussion of the correlation peaks. The SNR and pseudorange multipath oscillations are in phase with one another because when the multipath and direct signals are in phase, the multipath is at a maximum and when they are 180 degrees out of phase, the multipath is at a minimum. By inspecting the phasor diagram it can be seen that these same oscillations occur with the SNR measurement.

### 3.4 Multipath Phase

The multipath phase is affected by two phenomena. The first is the physical path delay of the signal, $\psi_{\text{path}}$, and the second is a change of the electromagnetic signal phase upon reflection, $\psi_{\text{reflect}}$. The two combine to form the total multipath relative phase, $\psi$. These effects are described in detail below.

#### 3.4.1 Path Delay

When a signal enters the antenna via an indirect route, the extra distance the signal travels as compared to a direct line of sight to the satellite is defined as path delay. For the
purposes of this study, all reflectors are modeled as perfectly conducting planar reflectors, for which the angle of reflection is equal to the angle of incidence. The path delay due to a planar reflector is illustrated in Figure 3.4. In this figure, \( \hat{n} \) is the normal vector of the plane, \( d \) is the normal distance from the plane reflector to the antenna, and \( \hat{e} \) is the unit line of sight vector from the user to the GPS satellite. The physical path delay is shown in the figure as the bold part of the reflected signal. An expression for the path delay, \( \psi_{\text{path}} \), is given in equation (3.3). Note in equation (3.3) that the path delay is only modeled as a function of the distance of the reflector from the antenna (\( d \)), the orientation of the reflector (\( \hat{n} \)), and the direction of the line of sight vector (\( \hat{e} \)), thus making it a purely spatially correlated phenomenon.

![Figure 3.4: Path delay for a reflected signal. The normal vector of the planar reflector is \( \hat{n} \), \( d \) is the normal distance from the plane reflector to the antenna, and \( \hat{e} \) is the unit line of sight vector from the user to the GPS satellite. The physical path delay is shown in the figure as the bold part of the reflected signal.](image)

\[
\psi_{\text{path}} = 2d(\hat{e} \cdot \hat{n})
\] (3.3)
In order to gain a better understanding of the dynamics of this problem, note that as a GPS satellite passes overhead, the relationship between the line of sight vector and the vector normal to the plane will change, causing the path delay to change. As the path delay increases or decreases, the multipath phasor rotates about the end point of the direct phasor. This rotation in turn causes the composite SNR and carrier phase magnitudes to oscillate sinusoidally as the path delay either increases or decreases, thus creating the correlated error found in the two signals.

3.4.2 Reflection of Polarized Signals

In order to discuss the reflection at oblique incidence, the plane of incidence must be defined. The plane of incidence is the plane formed by the unit vector normal to the reflecting surface and the incident line of sight vector [Balanis, 1989], as is illustrated below in Figure 3.5. Similarly, the plane of reflection is the plane formed by the vector normal to the reflector and the reflected line of sight vector. For the case of a perfect reflector, the plane of reflection is the same as the plane of incidence. For further illustration, a side view of the plane of incidence is shown in Figure 3.6. The direction of propagation of the incident and reflected waves are represented by the unit vectors \( \hat{\mathbf{u}}^i \) and \( \hat{\mathbf{u}}^r \), respectively. The incident wave has electric field components perpendicular to the direction of propagation and can be decomposed into two components, \( \hat{\mathbf{u}}^i_\parallel \) and \( \hat{\mathbf{u}}^i_\perp \), which are parallel and perpendicular to the plane of incidence.

\[
\vec{E}^i = E^i_\parallel \hat{\mathbf{u}}^i_\parallel + E^i_\perp \hat{\mathbf{u}}^i_\perp \tag{3.4}
\]

\[
\vec{E}^r = E^r_\parallel \hat{\mathbf{u}}^r_\parallel + E^r_\perp \hat{\mathbf{u}}^r_\perp \tag{3.5}
\]

Evaluated at the reflection point, the parallel and perpendicular Fresnel reflection coefficients are defined as follows:
\[
\Gamma_i = \frac{E'_i}{E''_i}, \quad \Gamma_\perp = \frac{E'_\perp}{E''_\perp}.
\] (3.6)

Figure 3.5: Illustration of the planes of incidence and reflection for a reflected signal. The incident and reflected signal unit vectors are denoted by the superscripts \(i\) and \(r\), respectively. The values \(\hat{u}_i\) and \(\hat{u}_r\) are the direction of propagation of the incident and reflected signals. The parallel and perpendicular electric field unit vectors are represented using the subscripts || and \(\perp\). The normal vector to the reflector is \(\hat{n}\).

Figure 3.6: Side view of the reflection frame. The incident and reflected signal unit vectors are denoted by the superscripts \(i\) and \(r\), respectively. The values \(\hat{u}_i\) and \(\hat{u}_r\) are the direction of propagation of the incident and reflected signals. The parallel and perpendicular electric field unit vectors are represented using the subscripts || and \(\perp\). The normal vector to the reflector is \(\hat{n}\).
In order to satisfy the boundary conditions for a perfect conductor, the incident and reflected components tangential to the surface must cancel at the reflection point [Stutzman, 1993]. Therefore, the reflection coefficients, $\Gamma_\parallel$ and $\Gamma_\perp$, must equal 1 and $-1$, respectively. The resulting parallel and perpendicular electric field components are shown in Figure 3.7.

Figure 3.7: Side view of the electric fields in the reflection frame. The incident and reflected electric fields are denoted by the superscripts $i$ and $r$, respectively. The values $\vec{u}^i$ and $\vec{u}^r$ are the direction of propagation of the incident and reflected signals. The parallel and perpendicular electric field components are represented using the subscripts $\parallel$ and $\perp$. The normal vector to the reflector is $\hat{n}$.

The fact that the perpendicular electric field component changes sign upon reflection in the plane of incidence adds complexity to the definition of the phase of the reflected signal. Because the circularly polarized signal is referenced to the antenna frame as defined by the spherical coordinates, the relationship between the plane of incidence frame and the antenna spherical coordinate frame takes on special significance.

In order to describe the behavior of the circularly polarized signal upon reflection, the transformations between the transmitted spherical frame and the plane of incidence frame must be included. $\alpha$. Because the radial component of the electromagnetic signal is zero for both of these reference frames, the transformation between the two frames is a simple rotation about the direction of propagation, and is represented by the angle, $\alpha$. Likewise, the angle
required to rotate from the plane of incidence (which is the same as the plane of reflection for a perfect conductor) frame to the antenna spherical frame is defined as $\alpha_i$. A graphical illustration of the rotations between the plane of incidence frame and the spherical coordinate frame for both the incoming signal and the reflected signal is shown for a general case in Figure 3.8.

![Diagram showing incident and reflected signals](image)

**Figure 3.8:** Example of the angle between the plane of incidence frame and the spherical frame. The vectors in the plane of incidence frame are represented using the subscripts $\parallel$ and $\perp$, while the spherical coordinate frame vectors are represented using the subscripts $\theta$ and $\phi$. The diagram on the left represents the rotation from the incident spherical coordinate frame to the plane of incidence frame, and the diagram on the right shows the opposite rotation. Note the sign of the rotation values for the two signals.

A mathematical representation of the rotations described in the previous paragraph is presented below. The incident $E_\theta$ and $E_\phi$ components are found by rotating from the circular polarized frame into the spherical coordinate frame, as shown in equation (2.6). For a perfectly RCP signal, the equation for the spherical coordinates reduces to:
\[
\begin{bmatrix}
E_\theta \\
E_\phi
\end{bmatrix}
= \frac{1}{\sqrt{2}} \begin{bmatrix}
\sin \phi_i + j \cos \phi_i \\
\cos \phi_i - j \sin \phi_i
\end{bmatrix} E_{RCP i}.
\] (3.7)

Then, the rotation from the spherical frame to the plane of incidence frame is

\[
\begin{bmatrix}
E_\perp \\
E_\parallel
\end{bmatrix}
= \begin{bmatrix}
\cos(\alpha_i) & \sin(\alpha_i) \\
-\sin(\alpha_i) & \cos(\alpha_i)
\end{bmatrix}
\begin{bmatrix}
E_\theta \\
E_\phi
\end{bmatrix}.
\] (3.8)

Upon reflection the parallel and perpendicular components flip in sign as follows [Stutzman, 1993]:

\[
\begin{bmatrix}
E_\perp \\
E_\parallel
\end{bmatrix}
= \begin{bmatrix}
-E_\perp \\
E_\parallel
\end{bmatrix}.
\] (3.9)

After reflection, the rotation from the plane of incidence frame to the spherical frame is

\[
\begin{bmatrix}
E_\theta \\
E_\phi
\end{bmatrix}
= \begin{bmatrix}
\cos(\alpha_r) & \sin(\alpha_r) \\
-\sin(\alpha_r) & \cos(\alpha_r)
\end{bmatrix}
\begin{bmatrix}
E_\perp \\
E_\parallel
\end{bmatrix},
\] (3.10)

and the final rotation from the reflected spherical frame into the circular polarized frame is

\[
\begin{bmatrix}
E_{RCP} \\
E_{LCP}
\end{bmatrix}
= \frac{1}{\sqrt{2}} \begin{bmatrix}
\sin \phi_r - j \cos \phi_r & \cos \phi_r + j \sin \phi_r \\
-\sin \phi_r + j \cos \phi_r & \cos \phi_r + j \sin \phi_r
\end{bmatrix}
\begin{bmatrix}
E_\theta \\
E_\phi
\end{bmatrix}.
\] (3.11)

Note that when a purely RCP signal reflects off of a perfectly conducting surface, the resulting signal is LCP because of the change in sign that occurs in the perpendicular and parallel components upon reflection. By expanding equations (3.7) through (3.11), the cancellation of the RCP component occurs. In order to determine the phase of the reflected signal in the circular polarized frame, the above process can be simplified by setting the incident RCP electric field value to a constant value with zero phase. The phase of the resulting LCP signal is the change in phase of the signal upon reflection. Combining the above equations, the reflected LCP signal can be defined in terms of the rotations between the incident and resulting spherical frame and the plane of incidence frame. The expansion and simplification of equations (3.7) through (3.11) are shown below.
\[ E_{\text{LCPR}} = \frac{1}{2} \left[ -\sin \phi + j \cos \phi \cos \alpha - j \sin \phi \left[ \begin{array}{c} \cos \alpha_r \\ \sin \alpha_r \\ -1 \\ 0 \end{array} \right] \right] E_{\text{RCP}} \] 

\[ E_{\text{LCPR}} = \left[ -\cos(\alpha_i - \alpha_r + \phi_i - \phi_r) + j \sin(\alpha_i - \alpha_r + \phi_i - \phi_r) \right] E_{\text{RCP}} \] 

The reflected LCP electric field is now given as the product of a complex scalar with unit magnitude and the incident RCP field. The net effect is thus a rotation of the incident RCP field to the resultant LCP electric field as is viewed by the antenna. The angular rotation that occurs is 

\[ \psi_{\text{reflect}} = \arctan \left( \frac{\sin(\alpha_i - \alpha_r + \phi_i - \phi_r)}{-\cos(\alpha_i - \alpha_r + \phi_i - \phi_r)} \right), \] 

where the quadrant for the phase is determined by the signs of the numerator and denominator. The angular rotation value is the phase shift of the signal as perceived by the antenna.

To illustrate the phase rotation, consider a simple case of the plane of incidence being aligned with the spherical coordinate frame. This case is equivalent to having a large, flat plate at some distance below the antenna. When the two frames are aligned, the rotation between the incident spherical coordinate frame and the plane of incidence frame is equal to the rotation between the plane of incidence frame and the reflected spherical coordinate frame, such that \( \alpha_i \) is equal to \( \alpha_r \). Also, when the two frames are aligned the incident azimuth is equal to the negative of the reflected azimuth. Note that the reference for the azimuth angle changes depending on whether the LCP or the RCP frame is being referenced. Therefore, in the case of an alignment between the circular polarized and the spherical frames, the phase shift of the signal upon reflection is 180 degrees. This finding is confirmed by Clark [1992] who has shown that the phase shift of the reflections from a flat ground are 180 degrees.

However, this represents only one specific case. What is not widely recognized is that all reflector orientations other than flat relative to the antenna do not cause the plane of
incidence and the antenna spherical coordinate frame to be aligned. Thus, for the case of an arbitrarily oriented reflector, the phase change of the signal upon reflection as viewed by the circularly polarized antenna will be some angle other than 180 degrees. This change is dependent on two things: a redirection of the signal causing a difference in the azimuth between the direct and the reflected signal and the phase shift of 180 degrees of the perpendicular component upon reflection. However, for most practical cases in which the reflector inclination angle is not too great, the azimuth of the incident and reflected fields will be close, and thus the phase shift can be approximated as 180 degrees.

3.5 Summary

This chapter presents the theoretical background behind multipath, including a discussion of the three GPS observables in which multipath can be found – the pseudorange, the carrier phase, and the signal-to-noise ratio. In addition, a theoretical basis for the multipath relative phase is derived. The multipath relative phase includes two contributions: the signal path delay and phase change upon reflection. Using the existing theory behind the reflection of polarized signals, an extensive development of the phase change upon reflection of a circularly polarized signal is presented.
CHAPTER 4

ALGORITHMS FOR MULTIPATH CORRECTION

4.1 Introduction

In this dissertation, several methods are implemented in order to identify and reduce carrier phase multipath. The one described in the first section is not a method for multipath reduction, \textit{per se}, but rather serves to isolate the multipath in the carrier phase measurement by reducing any additional phase error due to a mis-calibration of the line bias and baseline values. The remaining sections introduce algorithms specifically designed to reduce multipath in the signal, beginning with the second section which includes algorithms for reduction of phase residuals by applying a residual phase sky map. In the third section, the temporal SNR-based mitigation method developed by Comp [1996] is presented. The final section describes the algorithm for reflector identification using the SNR and spatial correlation.

4.2 Batch Estimation for Baseline and Line Bias Calibration

In order to use the residual phase measurements for attitude determination, the baselines and line bias values must be known in a body-fixed reference frame. An initial estimate of the baseline values can be computed \textit{a priori} using the mechanical specification of the spacecraft. However, because these values can change slightly once mounted onto the spacecraft or after launch, these values often times need significant calibration from the original measurements.
The calibration of the baselines and line biases can occur when an improved attitude reference is known, such as a solution from an IRU in spacecraft attitude determination applications. Calibration can also be applied when an platform is known to be stationary, such as in stationary ground experiments. The local-to-body rotation matrix, $^B{C}_L$, can then be used to express the measurement model for the differential phase:

$$\Delta \phi = (b^B \cdot e^B) - k + \beta + \nu = (b^B \cdot (^B{C}_L e^B)) - k + \beta + \nu,$$

(4.1)

where $b^B$ is the baseline vector in the body frame, $e$ is the line of sight vector, $k$ represents the integer ambiguity, $\beta$ is the line bias, and $\nu$ represents the noise on the signal and any additional errors that may occur, including multipath. Even though this expression is already linear about the baselines and line biases, it will be linearized about the a priori baselines and line biases from the mechanical specifications in order to avoid solving for the integer ambiguity, $k$. The ambiguity is computed directly using:

$$\bar{k} = \text{round}((b^B \cdot e^B) + \bar{\beta} - \Delta \phi),$$

(4.2)

where $\Delta \phi$ is the measured differential phase. Using the a priori line biases and baselines, the computed residual phase is then

$$\Delta \bar{\phi} = (b^B \cdot e^B) - \bar{k} + \bar{\beta}.$$

(4.3)

The linearized measurement model is defined by the difference between the measured and computed differential phase and is given in equation (4.4).

$$\delta \phi = \Delta \phi - \Delta \bar{\phi} = (\delta b^B \cdot e^B) - \bar{k} + \delta \beta + \nu$$

(4.4)

Taking the partials of the measurement model with respect to the baseline and line bias corrections, $\delta b$ and $\delta \beta$, the equations for the linear least squares problem are computed:

$$\delta \phi_i = H x_i,$$

(4.5)
where

$$
\phi_i = \begin{pmatrix}
\Delta \phi_i^1 \\
\Delta \phi_i^2 \\
\vdots \\
\Delta \phi_i^n
\end{pmatrix}, \quad
H = \begin{pmatrix}
e_\phi^1 & e_\phi^2 & e_\phi^3 & 1 \\
e_\phi^2 & e_\phi^3 & e_\phi^4 & 1 \\
\vdots & \vdots & \vdots & \vdots \\
e_\phi^n & e_\phi^{n+1} & e_\phi^{n+2} & 1
\end{pmatrix}, \quad
x_i = \begin{pmatrix}
\Delta b_{xi} \\
\Delta b_{yi} \\
\Delta b_{zi} \\
\Delta \beta_i
\end{pmatrix}, \quad \text{for } i = 1, 2, 3.
\tag{4.6}
$$

The subscript $i$ refers to the baseline number and the numbers 1 through $n$ refer to the satellites at each epoch. Note that the baselines and line bias estimates for each baseline are independent of the estimates for the other baselines. Also note that the measurement partial matrix, $H$, is not a function of the baseline, but only of the satellite line of sight vectors.

The corrections to the baseline vectors and line biases are computed by solving equation (4.5) in a least squares sense. If the initial estimates for the baselines and line biases are not accurate enough to ensure that the correct integer ambiguity, $k$, is computed, then the process is iterated until the integer values no longer change. Once the final baselines and line biases have been computed using a sufficiently large data set and an accurate attitude reference, the only errors which remain are due to measurement errors, such as receiver noise and multipath. In addition, errors will also occur in the signal if the baselines and line biases are changing over time, due to flexure in the spacecraft body or a change in the line bias due to temperature variations.

## 4.3 Algorithms for Sky Map Construction

If the majority of the baseline and line bias effects have been correctly removed from the phase measurement, and assuming that the time-varying baselines and line bias errors are small, the remainder of the error should be primarily due to spatially correlated multipath. Because multipath has been shown to be highly spatially correlated in other experiments [Cohen and Parkinson, 1991] [Irish et al., 1998], the same spatial correlation will be assumed here.
In order to construct the sky maps, the observables are sorted for each baseline independently, as a function of the line of sight vectors in the body-fixed reference frame. Thus, for each residual phase measurement, the corresponding azimuth and elevation of the incoming signal are computed from the three rectangular components of the line of sight vector as follows:

\[
az = \text{atan}2\left(\frac{e_y^B}{e_z^B}\right), \quad el = -\text{asin}\left(e_x^B\right)
\] (4.7)

Note that the \(x\)-component of the line of sight vector is down relative to the zenith angle. In order to represent the residuals as a function of the azimuth and elevation of the sign, several methods are developed: a spherical harmonic fit, a correction grid, and a two dimensional polynomial fit. These three methods are described in the sections below.

### 4.3.1 Spherical Harmonic Fit

The first method implemented is developed in Cohen and Parkinson [1991] and Cohen [1992]. Differential phase errors are modeled by a spherical harmonic approximation in the form:

\[
\delta\phi(el, az) = \sum_{l=1}^{n}\left[ J_l P_l(\cos el) + \sum_{m=1}^{l} P_{lm}(\cos el)(C_{lm} \cos(maz) + S_{lm} \sin(maz)) \right],
\] (4.8)

where \(P_{lm}\) are Legendre polynomials and \(n\) is the order of the spherical harmonic model. The coefficients of the spherical harmonic model are \(J_l\), \(C_{lm}\), \(S_{lm}\) and are determined by rearranging equation (4.8) into a set of normal equations and solving for the coefficients using a least squares fit to the residual phase errors. For a more complex reflective surface geometry, the order of the model, \(n\), is increased in order to account for the typically higher frequency of variation in the multipath that occurs with increased complexity. The order of the spherical harmonic fit is limited by the number of data points available.
4.3.2 Polynomial Fit

As an alternative model to the spherical harmonic fit, a two dimensional polynomial fit is used to represent the phase residuals. Here, the polynomial fit is not defined as a direct function of the azimuth and elevation angles. Making the fit a function of the azimuth and elevation would result in large function variations in the zenith direction, where the azimuth lines come very close together, and very small variations on the horizon, where the azimuth lines are much farther apart. Instead, the two dimensional polynomial is modeled as a function of the rectangular coordinates, $x$ and $y$, as follows:

$$x = (90 - el) \cos(az), \quad y = (90 - el) \sin(az).$$  \hfill (4.9)

Using these values, the model for the two dimensional polynomial fit is

$$\delta \phi(x, y) = A_0 + \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} x^i y^j,$$  \hfill (4.10)

where $n$ is the order of the polynomial fit. As with the spherical harmonic fit, the order of the model is increased to account for more complex reflector geometry, while being limited by the amount of data available.

4.3.3 Correction Grid

A third method is used to model the residual phase as a function of the line of sight vector. This simple method uses the variables in equation (4.9) to define a rectangular region. The region is then divided into a grid of bins of a certain size, defined as $\delta x$ by $\delta y$. Then, each residual phase measurement is placed into a bin corresponding to the $x$ and $y$ value for that signal. A phase correction value is then assigned to each bin by taking the mean of all of the values within a bin. If too few data points are available because spatial coverage is limited, the size of the bins may be increased to supply a better representation of
the data. The correction value for the individual data points is applied either by interpolation or as a table lookup.

4.4 Temporal SNR-based Multipath Correction

The temporal SNR-based multipath estimation procedure, developed by Comp [1996], determines differential phase corrections based upon variations in the SNR for the two antennas. Results using this method are presented in Axelrad et al. [1996] and Comp and Axelrad [1997]. It is an effective method for reducing specular multipath which is relatively small compared to the direct signal. The phasor diagram shown previously in Figure 3.3 illustrates the signal geometry for both a direct and a reflected signal.

The multipath error introduced into the GPS phase observation is modeled as

$$\delta\phi \approx \frac{\sum_{i=1}^{n} \alpha_i A_o \sin(\psi_i)}{A_o A_G + \sum_{i=1}^{n} \alpha_i A_o \cos(\psi_i)}$$  \hspace{1cm} (4.11)

and the corresponding SNR variation due to multipath is given by,

$$\delta\text{SNR} \approx \sum_{i=1}^{n} \alpha_i A_o \cos(\psi_i)$$  \hspace{1cm} (4.12)

where $A_o$ is the direct signal amplitude arriving at the antenna, $A_G$ is the antenna gain, $i$ is the index for each multipath constituent, and $n$ is the total number of constituents. The parameter $\alpha_i$ is the relative magnitude of the multipath signal relative to the direct and $\psi_i$ is the relative phase of the multipath relative to the direct.

As the satellite moves overhead each multipath constituent rotates with respect to the direct signal. This produces a time-correlated oscillation in both the composite SNR and differential phase with period related to the motion of the satellite across the sky and the distance to the reflector. The technique described in [Comp and Axelrad, 1997 and Comp, 1996] uses the SNR data as a function of the time along the satellite pass to adaptively
estimate the frequency, amplitude, and phase of several multipath constituents. Two algorithms are used – an adaptive notch filter for frequency estimation, and an adaptive least squares solution for the amplitude and phase of each multipath term. A phase correction profile is then constructed by combining the terms and is applied to the differential phase residuals.

An important advantage of the temporal SNR-based correction method is that it does not depend strongly on accurate differential phase predictions which require accurate knowledge of the spacecraft attitude. A drawback of this method is that it can take up to 100 data points per satellite pass to converge, and until the estimator has converged, reliable multipath estimates are not found. Thus, a significant amount of data goes uncorrected and it does not take advantage of the strong spatial correlation of multipath.

4.5 Reflector Identification Algorithm

The following sections describe a method to identify and reduce multipath by locating an effective reflector. First, the background for the algorithm is presented. The background includes a description of the method used to compute the antenna gain pattern and a discussion of the state variables used. Next, the initialization process is described, followed by two methods that can be used to identify an optimal state vector – a batch least squares estimation scheme and the Nelder-Mead batch algorithm.

4.5.1 Background

Prior to implementing the algorithm, the antenna gain value, $A_a$, is required to use the SNR for reflector identification. To find the antenna gain, the line of sight vectors are rotated from the local frame into the antenna frame using an a priori estimate for the spacecraft local-to-body attitude. Given the line of sight vectors in the antenna frame, the azimuth and
elevation of the GPS signal relative to the antenna are determined. The antenna gain for that satellite pass is then found by using a simple interpolation scheme as a function of the azimuth and elevation of the signal relative to the antenna frame. Note that in all of the evaluations done here, the antenna gain, $A_o$, is implemented in a normalized form and any additional amplitude values due to the gain are included in the estimation of $A_o$.

The state vector for the identification algorithm is:

$$x = [d, \text{az}, \text{el}, A_o, A_m]$$

(4.13)

The state is composed of five variables, where $d$ is the distance to the reflector, $\text{az}$ and $\text{el}$ are the azimuth and elevation of the plane normal vector, $A_o$ is the constant part of the direct signal amplitude in amplitude units, and $A_m$ is the multipath amplitude in amplitude units. The azimuth and elevation of the reflector are related to the plane unit normal vector by the following equation:

$$\hat{n} = [\sin(\text{el}) \cos(\text{el}) \cos(\text{az}) \cos(\text{el}) \sin(\text{el})],$$

(4.14)

where an elevation angle of 90 degrees is defined in the positive $x$-direction and an azimuth of 0 degrees is in the positive $y$-direction.

The task for the reflector identification algorithm is to find a state vector that minimizes the mean square residual between the computed and measured SNR. The process of reflector identification takes place in two steps. First, the state variables are initialized using a grid search method. Once the state variables have been initialized, the optimum reflector location is determined by using either the linearized least squares method or the Nelder-Mead algorithm.

4.5.2 Initialization

Before optimization of the state using either method can be performed using the SNR measurement data, the state variables must be initialized. Initialization is done in order to
avoid either a long convergence time or the possibility of identification of a false minimum. The initialization is accomplished by searching over a coarse three-dimensional grid containing the first three state values of distance, azimuth, and elevation, representing the full range of possible reflector locations. In order to search over the entire region of possibilities, the distance is allowed to grow as far away as a reflector is likely to be. In the case of a small spacecraft, this may only be a few meters. For ground-based tests, the distance may be as much as ten meters, where any distance larger than this would produce very high frequency errors which will likely appear as noise. The azimuth in the initialization search has a range of 0 to 360 degrees and the elevation is allowed to vary from 0 to 90 degrees. Keeping the elevation angle between this range always assures that the reflector is below the antenna, relative to the GPS satellites.

For each of the assumed locations and the GPS lines of sight for the satellite pass, a path delay profile is computed using equation (3.3). Note that if the line of sight vector drops behind the planar reflector, the path delay is set to zero and no multipath is considered for this region. An estimate for the constant part of the direct signal, $A_0$, is computed by performing a least squares fit to the SNR data as shown in equation (4.15).

$$SNR = A_0 A_d$$

(4.15)

Once the path delay and direct signal strength are known, an estimate of the multipath amplitude, $A_m$, is computed by solving equation (3.1) using least squares, where the composite amplitude is the measured SNR. After trying each of the possible reflector locations and computing the corresponding direct and multipath amplitudes, the location and amplitudes with the lowest SNR residual are used to initialize either the linearized least squares algorithm or the Nelder-Mead algorithm, for state optimization.
4.5.3 Nelder-Mead Algorithm

In the reflector identification algorithm, one method by which to optimize the state vector is the *fmins* function from MATLAB [Mathworks, 1992]. This function uses the Nelder-Mead simplex search scheme [Press et al., 1992]. For a state size of \( N \), a simplex is a geometrical shape formed by \( N+1 \) points, each defining a vertex of the figure. For a state size of two, the simplex is a triangle; for a state size of three, the simplex is a tetrahedron, etc. This search method does not require either gradients or other derivative information, and only requires function evaluations. It simply progresses over iteration steps where a new vertex for a state is defined in or near the current simplex. The function value is evaluated for the new vertex value and it replaces the current value if it reduces the overall volume of the simplex. This process continues until the diameter of the simplex is less than a certain, pre-defined tolerance. In MATLAB, the algorithm is terminated when either the increment size for the state or the residual change drops below some user-defined tolerance. Premature termination of the algorithm may occur if, during some anomalous step, the state or the residual fail to make significant progress, thus implying a false solution for the state. In order to test the solution for possible early termination, the state solution is used to re-initialize algorithm and the minimization process is evaluated again. If it is a false minimum, the step size for the state or residual will return above the tolerance value and the method will begin again.

4.5.4 Linearized Batch Estimation Algorithm

A linearized least squares batch estimation filter is also developed in this thesis as a means to identify an effective reflector location. The linearized batch algorithm utilizes the dependence on the reflector location in the multipath relative phase equation. Considered in this evaluation for the multipath relative phase are the effects due to path delay and a constant offset of 180 degrees to account for the phase shift of the circularly polarized signal. A phase
shift of 180 degrees is assumed in order to avoid the complexity involved with computing the partial derivatives of the reflection equations for circular polarization. Therefore, in this algorithm, it is assumed that the effect due to the apparent phase shift upon reflection of a circularly polarized signal contributes much less to the multipath relative phase than does the physical path delay.

After the state vector is initialized, the batch estimator is run. The measurement sensitivity is computed by taking partial derivatives of the measurement equation, defined in equation (3.1), with respect to the state variables. The measurement partial derivative matrix, $\mathbf{H}$, is defined by the following equations:

$$
\mathbf{H} = 
\begin{bmatrix}
\frac{\partial (A_x^2)}{\partial d}
& \frac{\partial (A_x^2)}{\partial az}
& \frac{\partial (A_x^2)}{\partial el}
& \frac{\partial (A_x^2)}{\partial A_o}
& \frac{\partial (A_x^2)}{\partial A_m}
\end{bmatrix}
$$

(4.16)

$$
\frac{\partial (A_x^2)}{\partial d} = -4 A_m A_o A_o (\hat{e} \cdot \hat{n}) \sin \psi
$$

$$
\frac{\partial (A_x^2)}{\partial az} = -4 A_m A_o A_o d (- e_x \cos el \sin az + e_z \cos el \cos az) \sin \psi
$$

$$
\frac{\partial (A_x^2)}{\partial el} = -4 A_m A_o A_o d (e_y \cos el \sin az + e_z \sin el \sin az) \sin \psi
$$

$$
\frac{\partial (A_x^2)}{\partial A_o} = 2 A_o A_o^2 + 2 A_m A_o \cos \psi
$$

$$
\frac{\partial (A_x^2)}{\partial A_m} = 2 A_m + 2 A_o A_o \cos \psi
$$

The measurement residual is the difference between the measured $\text{SNR}^2$ and that predicted from (3.1) using the initial estimate of the state. The update to the state, $\hat{x}$, is computed in a least squares sense using the measurement partial matrix and the measurement residual, $y$, as shown in equation (4.17).
\[ H^T \hat{H} \hat{\xi} = H^T \hat{y} \]  

(4.17)

After an update to the state is computed, it is added to the current estimate of the state, and this process is iterated until the state estimate converges.

4.5.5 Measurement Correction

Once the state vector has been computed for each antenna, carrier phase corrections are computed utilizing equation (3.1) for each satellite pass in which a reflector is identified. The differential phase correction is computed by differencing the corrections for the individual antennas. The differential phase corrections are then subtracted from the measured values and the updated differential phase measurements are filtered using an attitude estimation algorithm developed by Ward [1996].

4.6 Performance Evaluation

The performance of all of the algorithms presented here is tested by comparing the phase residuals before and after the phase corrections for an effective reflector have been applied. The root mean square (RMS) of the residuals is computed in order to assess the phase improvement. For attitude data, in addition to comparing the residuals, the resulting change in the attitude solution is noted.
CHAPTER 5

SIMULATION OF MULTIPATH USING THE NUMERICAL ELECTROMAGNETIC CODE - BASIC SCATTERING CODE

5.1 Introduction

Working with simulated data is a useful first step to test algorithms because the simulation testing environment is easy to control and manipulate, as compared to real experiments. Data described in this chapter were simulated using the Numerical Electromagnetic Code - Basic Scattering Code, a program developed at the Ohio State University, ElectroScience Laboratory [Marhefka and Silvestro, 1989]. The data are used to study multipath in simple reflecting environments using ideal antennas and to explore in detail the nature of what happens to the right-hand circularly polarized carrier wave as it interacts with reflective surfaces. Because this simulation software is independent of the reflector identification algorithm developed in this research, it is a useful tool to test the validity of the algorithm.

The purpose of this chapter is to describe how GPS SNR and carrier phase multipath data are simulated using the Numerical Electromagnetic Code - Basic Scattering Code (NEC-BSC). The sections which follow describe the simulation process by first discussing the input for both the antenna and reflector properties. Then the output data from the simulation routine are described along with the means by which these data are manipulated to simulate
the GPS SNR and differential carrier phase measurements. In the final section, four test cases are described and results for the reflector identification method are shown.

5.2 Background and Previous Work

The NEC-BSC [Marhefka and Silvestro, 1989] is a computer code used to analyze antenna radiation patterns in the presence of complex scattering structures for high frequency signals. The numerical analysis is based on the Uniform Theory of Diffraction (UTD). The version used for the simulation analysis, Version 3.0, is specifically designed to aid in the placement of antennas on the space station, which is comprised of many large plates and cylindrical structures. The NEC-BSC aids in determining the ideal placement of antennas by identifying periods of signal blockage and the levels of multipath and antenna to antenna interference.

In order to investigate multipath under complex reflector geometry, several researchers have proposed the use of diffraction modeling codes to simulate the electromagnetic signal environment on a spacecraft. The NEC-BSC was used by Gomez and others at NASA JSC to model the multipath environment on the International Space Station Alpha [Gomez et al., 1995] and for a GPS flight experiment in the Space Shuttle cargo bay [Gomez and Hwu, 1997]. The NEC-BSC was also used by Lippencott et al. [1996], to demonstrate a method to model both code and phase multipath for more generic spacecraft structures. Gomez and Hwu [1997] also discuss the limitations of this approach for modeling phase errors for small and complex structures.

5.3 Simulation Input

The input parameters describe the characteristics of the antenna and the geometry of the reflector and are discussed in detail below.
5.3.1 Antenna Input Data

The antenna input data is comprised of the antenna characteristics, the field regions to be included, and the satellite line of sight vectors. To define the antenna characteristics in the NEC-BSC, the user provides a location, the center frequency, and the antenna gain and phase patterns. The user specifies whether the output data will consist of both far and near field effects or if it will be limited to only far field assumptions. Finally, the lines of sight from the user antenna to the GPS satellite are also specified by entering the azimuth and zenith angles of the far field signals desired.

A complication arises in the NEC-BSC because all user antennas are treated as transmitters. Since a GPS antenna on-board a spacecraft would typically be acting as a receiving antenna, the reciprocity theorem is employed. The reciprocity theorem states that the radiation pattern of an antenna is the same whether it is receiving or transmitting [Balanis, 1997]. Therefore, in the simulations the signals are assumed to be transmitted from the user antenna at the GPS $L_f$ frequency (1575.42 MHz) and are received in the far field.

5.3.2 Reflector Input Data

The location and orientation of a reflecting surface are specified by listing the rectangular coordinates of the corners of the surface. A planar reflector can be specified as one of the following types: a transparent thin dielectric slab, a perfectly conducting metallic plate, a double-sided coated dielectric plate, or a one-sided coated dielectric plate. The user may also specify the types of scattering that will take place off of the reflecting surface. The kinds of scattering include specularly reflected fields, diffracted fields and any combination of the reflected and diffracted fields, including reflected-diffracted, diffracted-reflected, doubly reflected and doubly diffracted fields.
5.4 Output data

Two output fields are generated in order to model the multipath due to a reflector. One field contains all of the far field signals generated with no reflectors present and the second contains the signals with the reflectors present. The electric field output from the NEC-BSC is represented in spherical coordinates and the signals are produced for each azimuth and elevation point specified in the input file.

In order to simulate signals transmitted from the GPS satellites using reciprocity, the antenna transmits LCP and RCP signals as specified by the antenna phase and gain patterns. All of the signals in the far field are limited to being purely RCP, since the GPS satellites transmit a purely RCP signal. The signals are limited to only those which are RCP by discarding any signals which produce an LCP component in the far field. This is done by computing the projection of the output $E_\phi$ and $E_\theta$ components onto the RCP complex unit vector, shown in equation (2.4). Under the theorem of reciprocity, this process is equivalent to transmitting purely RCP signals at an antenna which receives both RCP and LCP components through the appropriate antenna patterns.

5.5 Simulation Setup

In the simulations, all of the multipath data are generated for a perfectly conducting planar reflector and only specular multipath is considered. Two antennas are used to simulate a single baseline on a spacecraft. The rectangular coordinates for these antennas are shown in Table 5.1. Note that in this rectangular coordinate frame, the x-direction is aligned with the antenna boresight direction.
Table 5.1: Antenna locations for the simulations.

<table>
<thead>
<tr>
<th>antenna</th>
<th>x (m)</th>
<th>y (m)</th>
<th>z (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>master</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>slave</td>
<td>0.05</td>
<td>0.7</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The antenna transmit pattern is generated by defining an amplitude profile of the RCP and LCP patterns, based on the pattern from a typical GPS antenna. The RCP and LCP total amplitude patterns, defined as the product of the antenna gain, $A_a$, and the direct signal strength, $A_{ds}$, are shown in Figure 5.1. In order to compute a normalized RCP gain pattern as is typically used in GPS applications, the total amplitude values are divided by the maximum amplitude of 26.2 AMU. Therefore, the direct signal strength, $A_{ds}$, is 26.2 AMU. The phases of all of the transmitted circular polarized signals are set to zero. The gain and phase values are then converted into complex notation and are rotated into the spherical coordinate frame using equation (2.6) so that they could be entered into the NEC-BSC. The GPS satellite pass considered begins at an azimuth angle of -105 degrees and a zenith angle of 90 degrees, passes directly over the sky at a zenith angle of 0 degrees, and sets at an azimuth angle of 75 degrees and a zenith angle of 90 degrees. The satellite pass is displayed graphically in Figure 5.2. Data for which the plate is blocking the transmission from the antenna to the far field are removed.
Figure 5.1: RCP and LCP gain patterns used in the simulation cases.

Figure 5.2: Satellite track for the simulation cases. The data points are plotted as a function of the azimuth and zenith angles.
Four cases using different reflector orientations were processed in the NEC-BSC. The tilt angle, direction of the plate, and normal distance from the master antenna to the reflector, represented as \( d \), are listed below in Table 5.2. The corresponding rectangular coordinates of the corners of the planar reflector, as specified in the input file for the NEC-BSC, are listed in Table 5.3. The diagram of the plate geometry, relating the plate location and orientation and the tilt angle, azimuth, and distance is shown in Figure 5.3. The vector normal to the planar reflector is \( \hat{n} \). The azimuth is the angle between the \( x \)-direction and the projection of the normal vector onto the \( x-y \) plane. The tilt angle is the angle between the normal vector and the \( z \)-direction.

Table 5.2: Orientation of the planar reflector for the four cases.

<table>
<thead>
<tr>
<th>case</th>
<th>( d ) (meters)</th>
<th>azimuth (degrees)</th>
<th>tilt angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.500</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.4924</td>
<td>90.0</td>
<td>10.0</td>
</tr>
<tr>
<td>3</td>
<td>0.3625</td>
<td>90.0</td>
<td>43.53</td>
</tr>
<tr>
<td>4</td>
<td>0.3625</td>
<td>40.0</td>
<td>43.53</td>
</tr>
</tbody>
</table>

Table 5.3: Corners of the planar reflector for the four cases.

<table>
<thead>
<tr>
<th>case</th>
<th>corner #1 (meters)</th>
<th>corner #2 (meters)</th>
<th>corner #3 (meters)</th>
<th>corner #4 (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1000, 1000, -0.5)</td>
<td>(-1000, 1000, -0.5)</td>
<td>(-1000, -1000, -0.5)</td>
<td>(1000, -1000, -0.5)</td>
</tr>
<tr>
<td>2</td>
<td>(1000, 1000, -176.5)</td>
<td>(-1000, 1000, -176.5)</td>
<td>(-1000, -1000, 175.5)</td>
<td>(1000, -1000, 175.5)</td>
</tr>
<tr>
<td>3</td>
<td>(1000, 1000, -950.5)</td>
<td>(-1000, 1000, -950.5)</td>
<td>(-1000, -1000, 949.5)</td>
<td>(1000, -1000, 949.5)</td>
</tr>
<tr>
<td>4</td>
<td>(1409, -123, -950.5)</td>
<td>(123, 1409, -950.5)</td>
<td>(-1409, 123, 949.5)</td>
<td>(-123, -1409, 949.5)</td>
</tr>
</tbody>
</table>
The SNR and residual phase data are generated for each of the four cases using the complex representations shown in equations (2.8) and (2.9), respectively. The antenna gain values, $A_o$, are generated by using the zenith angles of the direct signal to interpolate to the zenith angles of the gain values shown in Figure 5.1. Note that to compute the antenna gain, the RCP pattern shown in Figure 5.1 is normalized by the maximum gain, 26.2 AMU.

For these simulations, the location of the large planar reflector and the amplitude of the direct signal, $A_o$, are constant throughout. However, the multipath amplitude value, $A_m$, is not constant over the data pass because the multipath amplitude is dependent on the gain of the transmitted LCP signal. The amplitude of the LCP signal varies due to the antenna gain as a function of the zenith angle, as shown in Figure 5.1.
5.6 Results

The reflector identification algorithm is implemented for the four sets of data with an initial state estimate error as shown below:

\[
\begin{align*}
\delta d & = 3 \text{ centimeters} \\
\delta az & = 2 \text{ degrees} \\
\delta el & = 2 \text{ degrees} \\
\delta A_o & = 4 \text{ AMU} \\
\delta A_m & = 1 \text{ AMU}
\end{align*}
\]  

(5.1)

Using the identification method, the reflector locations are found for both the master and slave antenna. In order to refine the state estimate, the Nelder-Mead algorithm is used, rather than the batch least squares algorithm, so that contributions due to the phase shift of the signal upon reflection can be easily included. As an illustration, Figure 5.4 shows the contributions of the path delay and the phase shift upon reflection to the multipath relative phase for Cases 1 and 4. This figure shows that the phase shift of the signal upon reflection as perceived by the antenna is exactly one half cycle for Case 1, where the planar reflector is exactly flat beneath the antenna. For Case 4, where the reflector is inclined at 44 degrees, the phase offset due to the reflection deviates slightly from 180 degrees.
Figure 5.4: Multipath relative phase contributions for (a) case 1 and (b) case 4.
With the exception of the first case, all five states are estimated using the reflector identification algorithm. In Case 1, the ambiguity of the reflector azimuth due to the plate being flat caused difficulty in identifying a reflector. So, instead of using the algorithm to determine all five states, the azimuth and elevation are held fixed to 0 degrees and 90 degrees, respectively, while the reduced state, including only the distance to the reflector and the two amplitude values, is optimized. Note that an elevation angle of 90 degrees corresponds to a tilt angle of 0 degrees.

The simulated and estimated values for the SNR for the two antennas and for the residual differential phase for the baseline is shown for each case in the figures below. Table 5.4 lists the reflector locations and the multipath and direct amplitudes identified for each case. Table 5.5 shows the root mean square of the residuals before and after the SNR and residual differential phase values are applied for the identified reflectors.
Figure 5.5: Simulated and estimated SNR for simulation case 1 – a flat planar reflector located 50 centimeters below the master antenna. The top figure is for the master antenna and the bottom is for the slave. The gray line is the magnitude of the simulated signal and the black is the estimated value.
Figure 5.6: Simulated and estimated residual differential phase for simulation case 1 – a flat planar reflector located 50 centimeters below the master antenna. The gray line is the simulated residual differential phase and the black is the estimated value.
Figure 5.7: Simulated and estimated SNR for simulation case 2 – a planar reflector inclined at 10 degrees and 49 centimeters below the master antenna. The top figure is for the master antenna and the bottom is for the slave. The gray line is the magnitude of the simulated signal and the black is the estimated value.
Figure 5.8: Simulated and estimated differential phase for simulation case 2 – a planar reflector inclined at 10 degrees and 49 centimeters below the master antenna. The gray line is the simulated residual differential phase and the black is the estimated value.
Figure 5.9: Simulated and estimated SNR for simulation case 3 – a planar reflector inclined at 44 degrees, 36 centimeters below the master antenna, and rotated by an azimuth of 90 degrees. The top figure is for the master antenna and the bottom is for the slave. The gray line is the magnitude of the simulated signal and the black is the estimated value.
Figure 5.10: Simulated and estimated differential phase for simulation case 3—a planar reflector inclined at 44 degrees, 36 centimeters below the master antenna, and rotated by an azimuth of 90 degrees. The gray line is the simulated residual differential phase and the black is the estimated value.
Figure 5.11: Simulated and estimated SNR for simulation case 4 – a planar reflector inclined at 44 degrees, 36 centimeters below the master antenna, and rotated by an azimuth of 40 degrees. The top figure is for the master antenna and the bottom is for the slave. The gray line is the magnitude of the simulated signal and the black is the estimated value.
Figure 5.12: Simulated and estimated differential phase for simulation case 4 – a planar reflector inclined at 44 degrees, 36 centimeters below the master antenna, and rotated by an azimuth of 40 degrees. The gray line is the simulated residual differential phase and the black is the estimated value.

Table 5.4: State estimates for the four simulation cases.

<table>
<thead>
<tr>
<th>case</th>
<th>antenna</th>
<th>$d$ (meters)</th>
<th>$az$ (degrees)</th>
<th>$el$ (degrees)</th>
<th>$A_o$ (AMU)</th>
<th>$A_m$ (AMU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>master</td>
<td>0.5000</td>
<td>0.0</td>
<td>90.0</td>
<td>26.18</td>
<td>1.92</td>
</tr>
<tr>
<td></td>
<td>slave</td>
<td>0.5000</td>
<td>0.0</td>
<td>90.0</td>
<td>26.22</td>
<td>1.92</td>
</tr>
<tr>
<td>2</td>
<td>master</td>
<td>0.4924</td>
<td>90.9</td>
<td>80.0</td>
<td>26.18</td>
<td>1.91</td>
</tr>
<tr>
<td></td>
<td>slave</td>
<td>0.4933</td>
<td>90.4</td>
<td>80.0</td>
<td>26.18</td>
<td>1.92</td>
</tr>
<tr>
<td>3</td>
<td>master</td>
<td>0.3634</td>
<td>91.9</td>
<td>46.3</td>
<td>26.22</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>slave</td>
<td>0.3609</td>
<td>89.8</td>
<td>46.5</td>
<td>26.22</td>
<td>1.68</td>
</tr>
<tr>
<td>4</td>
<td>master</td>
<td>0.3732</td>
<td>36.3</td>
<td>44.8</td>
<td>26.20</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>slave</td>
<td>0.3691</td>
<td>43.9</td>
<td>47.8</td>
<td>26.18</td>
<td>1.64</td>
</tr>
</tbody>
</table>
Table 5.5: Measurement errors for the four simulation cases.

<table>
<thead>
<tr>
<th>case</th>
<th>antenna</th>
<th>RMS of SNR (AMU)</th>
<th>RMS of $\delta \Delta \phi$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>before</td>
<td>after</td>
</tr>
<tr>
<td>1</td>
<td>master</td>
<td>17.61</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>slave</td>
<td>18.09</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>master</td>
<td>17.90</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>slave</td>
<td>17.95</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>master</td>
<td>19.75</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>slave</td>
<td>19.81</td>
<td>0.17</td>
</tr>
<tr>
<td>4</td>
<td>master</td>
<td>19.62</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>slave</td>
<td>19.22</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Overall, the reflector identification algorithm works very well in finding reflector locations for the simulation cases. In Case 1, the distance to the reflector differs from the truth value by less than 0.1 millimeters. Such precise results are most likely found because of the reduced state size of three for the flat-plate case, rather than a size of five as in the other cases. Cases 2 and 3 produce distance results that differ from the truth value by 1 to 3 millimeters. Case 4 produces results with an error of approximately 10 millimeters in distance.

A similar trend is seen when viewing the results of the reflector orientation. Note that in the results in Table 5.4, the elevation angle (90 degrees minus the tilt angle) is reported. The angular results for Case 2 agree with the truth angles by approximately 0.9 degrees in azimuth and better than 0.01 degrees in elevation. Similar results are seen for Case 3 and 4, where the azimuth differs from the truth by approximately 3 degrees and the elevation differs by more than 4 degrees.

The results shown in Table 5.4 for the reflector location directly correspond to the plots of the simulated and estimated SNR values displayed in Figure 5.5 through Figure 5.12. The plot of the SNR fit for Cases 1 and 2 follow the truth value very closely, while the plots of the SNR fit for Case 3 and 4 indicate slightly more error in the fit. The same is true for the
plots of the differential phase for Cases 1 and 2, which match the simulated values very well, while the match for Case 3 and Case 4 deviate from the simulated values slightly more.

The residual improvements for each of the cases is shown in Table 5.5 for the reflectors identified. In all four of the cases, the residual in the SNR is reduced by over 99%, indicating a very good match between the simulated and the estimated SNR. The improvement in the RMS of the phase follows the same trend as the accuracy of the identified reflector. The cases in which the reflector location more closely matches the truth value, Cases 1 and 2, provide greater residual phase improvement than in the latter two cases. For the first two cases, the residual improvement in the phase is better than 94 percent for each, while the improvement shown in the phase for the last two cases is approximately 83 percent. The error for the simulations is primarily caused by the varying amplitude of the true multipath, while only a constant multipath amplitude is estimated for the entire set of data.

5.7 Summary and Conclusions

GPS carrier phase and SNR data are simulated using the NEC-BSC. This chapter outlines the procedure for generating these data, including the electric field rotations between a spherical coordinate frame and a circular polarized reference frame. Four sets of data are simulated with each using a different orientation of a single plate reflector.

In all of the cases, a reflector is found such that the differential phase residuals are reduced by 83 to 94 percent. These results are found by estimating constant values for the reflector location and direct and multipath amplitudes. The small error remaining in the phase and SNR measurements is attributed to the fact that the multipath amplitude varies over time, rather than being a constant value, as is estimated. With the overall phase reduction shown, the assumption of constant multipath amplitude seems to be acceptable.
The simulations proved to be very useful in understanding the nature of GPS carrier phase multipath and in testing the ability of the reflector identification algorithm. The model for how multipath affects the SNR and the relationship between the SNR and carrier phase multipath are validated by processing the simulated data and computing accurate results for the reflector location and the amplitudes of the direct and reflected signals.
CHAPTER 6

GROUND EXPERIMENTS

6.1 Introduction

Once initial testing of the reflector identification algorithm is done using simulated GPS data, the tests can be taken one step further by generating GPS carrier phase and SNR multipath data using real antennas and receivers, but in a controlled environment. The test for the data chosen here is a set of ground experiments where the location of the antennas is precisely known and the multipath is induced intentionally.

The following sections in this chapter begin by describing the previous work done with these data and the experimental setup, including the location of the antennas and the type of receiver used and a description of the surrounding environment. Then, the reflector identification algorithm is applied to these data and results are shown.

6.2 Previous Work

On October 7, 1995, an experiment was performed by C. Behre at a Naval Research Laboratory test facility in order to collect data in a controlled environment. A mockup of the JAWSAT spacecraft was mounted on top of a spin table. The experiment was composed of two segments – one with the table stationary and one with it rotating. The data taken while the platform was rotating were used in testing algorithms for GPS carrier phase-based spin axis estimation [Behre, 1997]. Additionally, the static data were processed by C. Comp for
multipath mitigation using temporal-based SNR model [Comp, 1996]. Using the temporal SNR model to evaluate data from the number one baseline for satellite 17, Comp showed a successful reduction of the 99.7 percentile phase multipath from 24.6 to 19.6 millimeters, where the average $3\sigma$ noise is 11.7 millimeters. However, this method did not utilize the spatial dependence of the observed multipath and the \textit{a priori} knowledge of the likely reflecting surfaces. In order to take advantage of these dependencies, the satellite data are processed using the reflector identification algorithm and the results for this are displayed in the remainder of this chapter.

6.3 Experimental Setup

A Trimble TANS Vector attitude receiver was used for the testing [Cohen, 1992], [Trimble, 1993]. Three Trimble patch antennas, a master and two slaves, were affixed to a JAWSAT satellite mockup mounted upon a three-axis rotational spin table. A 0.6 meter by 0.6 meter square aluminum plate reflector, inclined at 45 degrees was mounted to the corner in which the number 2 slave would normally be affixed. During the experiment, 90 minutes of static data were collected. Upon evaluation the data are found to show very clear signs of multipath, both in the carrier phase and in the SNR. A photograph of the experimental setup is shown in Figure 6.1.
Figure 6.1: Ground test experimental setup (Courtesy of C. Behre).

As is shown in the figure, the antennas are mounted on a spin table, and are located approximately 1.5 meters above a concrete surface. The antenna locations were found by doing a self survey of the antenna setup. This procedure was done by C. Behre and is described in Comp [1996]. The initial estimate of the antenna locations, specified in a GPS antenna body frame, are shown in Table 6.1. Then, the attitude of the JAWSAT mockup was computed using the TANS Vector receiver and is used to rotate the antenna locations into the orbit-local frame. The mean attitude solution, represented in Euler angles, from the TANS Vector is shown below:

\[
\begin{align*}
roll &= 2.3 \text{ degrees} \\
pitch &= -2.0 \text{ degrees} \\
yaw &= -268.9 \text{ degrees}
\end{align*}
\]

(6.1)

The baselines are rotated by these angles from the body frame into the local frame and the resulting baseline lengths are listed in Table 6.2. Note that in this reference frame the x-
direction is in the zenith direction and the master antenna is assumed to be located at the origin of the reference frame. The line bias values for each of the baselines are also shown here. Additionally, the line of sight vectors from the antenna array to the GPS satellites are computed by using the position of the antennas as reported by the receiver and the precise GPS ephemerides as reported by JPL [Zumberge and Bertiger, 1996].

Table 6.1: Antenna body frame locations on the JAWSAT mockup.

<table>
<thead>
<tr>
<th>antenna</th>
<th>$x$ (meters)</th>
<th>$y$ (meters)</th>
<th>$z$ (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>master</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>slave 1</td>
<td>0.0033</td>
<td>-0.3492</td>
<td>0.2691</td>
</tr>
<tr>
<td>slave 3</td>
<td>0.0011</td>
<td>0.3454</td>
<td>0.4468</td>
</tr>
</tbody>
</table>

Table 6.2: Antenna locations in the local frame on the JAWSAT mockup.

<table>
<thead>
<tr>
<th>baseline</th>
<th>$x$ (meters)</th>
<th>$y$ (meters)</th>
<th>$z$ (meters)</th>
<th>line bias (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0079</td>
<td>-0.2649</td>
<td>-0.3523</td>
<td>0.838</td>
</tr>
<tr>
<td>3</td>
<td>-0.0282</td>
<td>-0.4506</td>
<td>0.3393</td>
<td>0.350</td>
</tr>
</tbody>
</table>

6.4 Results

Before the ground test data are processed, improved baseline and line bias solutions are computed for the two baselines using the batch line bias and baseline estimation scheme presented in Chapter 4. In order to provide more accurate results, residuals over a limit of one tenth of a cycle are removed from the processing after the first iteration. The new baseline and line bias solutions are shown in Table 6.3 and are the values used to calculate the differential carrier phase residuals.
Table 6.3: Improved antenna locations in the local frame on the JAWSAT mockup

<table>
<thead>
<tr>
<th>baseline</th>
<th>$x$ (meters)</th>
<th>$y$ (meters)</th>
<th>$z$ (meters)</th>
<th>line bias (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0127</td>
<td>-0.2632</td>
<td>-0.3529</td>
<td>0.813</td>
</tr>
<tr>
<td>3</td>
<td>0.0033</td>
<td>-0.4492</td>
<td>0.3414</td>
<td>0.229</td>
</tr>
</tbody>
</table>

Three out of the seven total satellite passes taken during the 90 minute time period are evaluated using the reflector identification algorithm. Only three were chosen because the other four passes did not display significant multipath in either the SNR or the residual differential phase. The satellites chosen are GPS PRN 17, 20, and 23, with data sets lasting approximately 61 minutes, 22 minutes, and 30 minutes, respectively. The line of sight vectors for the three satellite passes are displayed in Figure 6.2 in a polar plot as a function of the azimuth and zenith angle. The reflector identification algorithm is implemented using the raw SNR data and the antenna gain data for the Trimble patch antenna shown in Figure 2.2, provided by R. Allshouse of Allied Signal and NASA Goddard Spaceflight Center [Comp, 1996].
Figure 6.2: Tracks of the three satellites used in the ground experiments. The tracks are plotted as a function of the azimuth and zenith angle of the data points.

The reflector locations identified for each satellite/antenna combination are summarized in Table 6.4. Table 6.5 shows the SNR fit for the reflector locations, and Table 6.6, indicates the phase improvements for each of the baselines using the reflectors identified. Plots of the computed and measured SNR and residual phase values for these reflectors are shown in Figures 6.3 through 6.8.
Table 6.4: Reflector locations for the ground test.

<table>
<thead>
<tr>
<th>satellite</th>
<th></th>
<th></th>
<th>azimuth (degrees)</th>
<th>elevation (degrees)</th>
<th>$A_0$ (AMU)</th>
<th>$A_m$ (AMU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 master</td>
<td>1.6618</td>
<td>0.01</td>
<td>72.98</td>
<td>31.07</td>
<td>1.470</td>
<td></td>
</tr>
<tr>
<td>slave 1</td>
<td>1.6947</td>
<td>7.47</td>
<td>73.46</td>
<td>22.961</td>
<td>2.216</td>
<td></td>
</tr>
<tr>
<td>slave 3*</td>
<td>1.3031</td>
<td>237.70</td>
<td>33.24</td>
<td>19.766</td>
<td>2.480</td>
<td></td>
</tr>
<tr>
<td>20 master</td>
<td>1.4370</td>
<td>34.25</td>
<td>83.03</td>
<td>23.503</td>
<td>0.513</td>
<td></td>
</tr>
<tr>
<td>slave 1</td>
<td>1.2165</td>
<td>89.89</td>
<td>70.10</td>
<td>27.705</td>
<td>1.145</td>
<td></td>
</tr>
<tr>
<td>slave 3</td>
<td>1.4078</td>
<td>54.93</td>
<td>78.91</td>
<td>26.979</td>
<td>1.041</td>
<td></td>
</tr>
<tr>
<td>23 master*</td>
<td>0.3394</td>
<td>310.02</td>
<td>62.80</td>
<td>28.330</td>
<td>2.541</td>
<td></td>
</tr>
<tr>
<td>slave 1</td>
<td>1.7190</td>
<td>31.07</td>
<td>85.92</td>
<td>27.702</td>
<td>1.686</td>
<td></td>
</tr>
<tr>
<td>slave 3</td>
<td>1.7522</td>
<td>353.86</td>
<td>75.88</td>
<td>17.080</td>
<td>1.436</td>
<td></td>
</tr>
</tbody>
</table>

* Solution and initialization do not match well.

Table 6.5: SNR fits for the three ground test cases.

<table>
<thead>
<tr>
<th>satellite</th>
<th>antenna</th>
<th>RMS of SNR (AMU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 master</td>
<td>before</td>
<td>15.889</td>
</tr>
<tr>
<td>slave 1</td>
<td>11.634</td>
<td>1.382</td>
</tr>
<tr>
<td>slave 3</td>
<td>10.173</td>
<td>1.171</td>
</tr>
<tr>
<td>20 master</td>
<td>7.391</td>
<td>0.488</td>
</tr>
<tr>
<td>slave 1</td>
<td>8.605</td>
<td>0.682</td>
</tr>
<tr>
<td>slave 3</td>
<td>8.462</td>
<td>0.591</td>
</tr>
<tr>
<td>23 master</td>
<td>10.596</td>
<td>0.741</td>
</tr>
<tr>
<td>slave 1</td>
<td>10.217</td>
<td>1.426</td>
</tr>
<tr>
<td>slave 3</td>
<td>6.526</td>
<td>0.789</td>
</tr>
</tbody>
</table>

Table 6.6: Phase improvements for the three ground test cases.

<table>
<thead>
<tr>
<th>satellite</th>
<th>baseline</th>
<th>RMS of $\delta\Delta\phi$ (mm)</th>
<th>Improvement (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 1</td>
<td>12.01</td>
<td>9.47</td>
<td>21.1</td>
</tr>
<tr>
<td>20 1</td>
<td>22.66</td>
<td>26.94</td>
<td>-18.9</td>
</tr>
<tr>
<td>3 1</td>
<td>11.81</td>
<td>11.69</td>
<td>1.0</td>
</tr>
<tr>
<td>3 1</td>
<td>15.47</td>
<td>15.40</td>
<td>0.5</td>
</tr>
<tr>
<td>23 1</td>
<td>8.45</td>
<td>6.58</td>
<td>22.1</td>
</tr>
<tr>
<td>3 1</td>
<td>9.13</td>
<td>7.92</td>
<td>13.3</td>
</tr>
</tbody>
</table>
Figure 6.3: Measured and computed SNR for the three antennas, satellite 17. The top figure is for the master antenna, the middle for the number 1 slave and the bottom for the number 3 slave. The measured SNR values are shown in gray and the computed values in black.
Figure 6.4: Residual differential phase and computed phase corrections for satellite 17. The top figure is for the number 1 baseline and the bottom is for the number 3 baseline. The measured quantities are in gray and the computed in black.
Figure 6.5: Measured and computed SNR for the three antennas, satellite 20. The top figure is for the master antenna, the middle for the number 1 slave and the bottom for the number 3 slave. The measured SNR values are shown in gray and the computed values in black.
Figure 6.6: Residual differential phase and computed phase corrections for satellite 20. The top figure is for the number 1 baseline and the bottom is for the number 3 baseline. The measured quantities are in gray and the computed in black.
Figure 6.7: Measured and computed SNR for the three antennas, satellite 23. The top figure is for the master antenna, the middle for the number 1 slave and the bottom for the number 3 slave. The measured SNR values are shown in gray and the computed values in black.
Figure 6.8: Residual differential phase and computed phase corrections for satellite 23. The top figure is for the number 1 baseline and the bottom is for the number 3 baseline. The measured quantities are in gray and the computed in black.
The method works very well overall in identifying effective reflectors by which to model multipath in the SNR and the carrier phase. Most notably, the residual differential phase results for baseline 1 for satellite 17 and both of the baselines for satellite 23 very closely follow the trends of the true residual phase and only deviate somewhat during the first portion of the data.

The possible sources of error in the results fall into four categories. The first is a mis-modeling of the data due to the existence of another reflection source closer to the antennas. The second possible source of error is caused by estimating the multipath amplitude as a constant, when the true amplitude varies as a function of time and line of sight direction. Another type of error that could exist in the signal is an error in the calculation of the true residual phase error. A final cause for error could be due to the short data arcs analyzed. Each of these error sources are described in detail below.

The results shown in Table 6.4 indicate that the primary effective reflector location found for each of the antennas corresponds to the ground. However, in two of the cases, the number 3 slave antenna viewing satellite 17 and the master antenna viewing satellite 23, do not appear to find the ground as the primary reflector. In these cases, the reflector is closer to the antennas and tilted at elevation angles not as nearly flat as the rest of the data sets. These reflectors correspond to lower frequency multipath and do not fit to the higher frequency multipath as shown in all of the other cases. Additionally, in these two data sets, the results provided by the $f_{mins}$ function in MATLAB do not closely match the initial value given to $f_{mins}$, where the initial and final distance values differ by 26 and 69 centimeters for satellites 17 and 29, thus indicating a poor quality solution. In all of the other cases, the initial and final distance values before and after $f_{mins}$ is applied differs by less than a few centimeters. The cause for these errors could be that a second reflector closer to the antennas may need to be estimated. The second reflector causes a low frequency multipath. The estimation of an extra reflector is possibly needed in the master antenna of the data pass of satellite 17. This
could also be the cause of the erroneous results in the number 3 slave for satellite 17 and the master antenna for satellite 23, in that the algorithm is identifying a reflector causing low frequency multipath rather than one farther away.

Errors between the computed and true residual phase could also be due to the fact that the multipath amplitude is modeled as a constant value, when it is clearly shown in the NEC-BSC simulations that the multipath amplitude will vary as a function of the LCP gain pattern. However, from the simulation runs, the amplitude variations do not seem to affect the results on too large of a scale. Errors due to mis-modeling of the multipath amplitude variation may be causing some of the errors in baseline 1 of satellite pass number 17 and both of the baselines of satellite pass number 23.

Another cause of error could be a mis-modeling in the truth phase error, which is directly affected by baseline or line bias errors. The large deviation of up to 70 millimeters (~0.35 cycles) in the residual phase for baseline number 3 in Figure 6.4, indicates that there is an error other than multipath in the phase. Based on the phasor diagram shown in Figure 3.3, carrier phase multipath larger than one quarter of a cycle (approximately 5 centimeters) could only be caused by a reflected signal that is stronger than the direct, which is unlikely in this experiment. This points to an error in the residual phase computation, thus making it impossible to correct the large deviation with the multipath correction scheme presented here. The cause for the large residual in this baseline is not known, and does not appear to occur to this extreme in any of the other ground data sets.

Finally, the error in the results could be caused by the lack of longer periods of multipath data. The receiver collected data for less than 90 minutes, which does not allow the satellites enough time to pass over a large portion of the sky. With more data, the reflector locations would be more likely to accurately represent multipath in both the SNR and the carrier phase because short-period error sections, such as the one for baseline 3 in Figure 7.7, would not dominate the data sets.
6.5 Conclusions

Overall, the reflector identification algorithm worked well on the ground test data in locating reflectors that match the SNR profiles. The reflectors found are shown to improve the phase residuals in most of the data sets where the ground is identified as the primary source of multipath.

The results presented in this chapter indicate the ability of the algorithm to determine a reflector location that successfully reduces the phase error due to multipath for a static ground test. This implies that the reflector identification algorithm can be useful in reducing multipath in ground station applications such as these.

The error in the residual phase is believed to result from four possible causes. First, the multipath amplitude is being modeled as a constant value. Second, the short length of the data sets limits the ability of the reflector identification algorithm because there is less information available to form an accurate solution. Also, estimating the multipath amplitude using adaptive methods would improve these results, since it is clear in both the SNR and phase plots that the multipath amplitude does vary. Additionally, there is a lower frequency oscillation in the master and the number 3 slave antennas that is not being accurately modeled by the single large reflector approximately 1.5 meters away. Estimation of a second effective reflector, which is possibly the square aluminum plate, would likely also decrease the differential phase residuals.
CHAPTER 7

CRISTA-SPAS CASE STUDY

7.1 Introduction

CRISTA-SPAS was one of the first experiments in GPS attitude determination [Brock et al., 1995]. Since the satellite also had a star tracker capable of an attitude accuracy of 0.05 degrees, it served as an ideal means by which to check the attitude accuracy of the GPS receiver [Ward, 1996]. The SNR and phase data taken from the Trimble TANS Vector receiver are used in order to test the algorithms developed in this thesis for GPS carrier phase multipath reduction in a setting for removing multipath for improved attitude determination accuracy. These data appear to be an ideal test for the identification algorithm because of the large errors in the data, believed to be caused by multipath [Ward, 1996 and Comp, 1996].

The remainder of the chapter begins with a brief overview of the CRISTA-SPAS mission and the calibration of the baselines and line biases using the batch scheme presented in Chapter 4. Then in the next section, the GPS SNR and residual carrier phase data are analyzed for spatial repeatability. This is an important characteristic because, in order to use the reflector algorithm, the data must show spatial dependence. Once repeatability has been verified, the spatial sky map method is implemented and attitude solution improvements are presented. Also, phase corrections are computed using the temporal SNR-based multipath mitigation technique developed by Comp [1996] are applied to the CRISTA-SPAS phase data and the new attitude solutions are presented. Finally, the results found using the correction
algorithms for GPS carrier phase multipath using the SNR and spatial repeatability are presented.

7.2 CRISTA-SPAS Background

CRISTA-SPAS (CRyogenic Infrared Spectrometers and Telescopes for the Atmosphere – Shuttle Pallet Satellite) was successfully launched on November 3, 1994 with the STS-66 Space Shuttle Atlantis. The crew of Atlantis deployed the CRISTA-SPAS satellite and it obtained data in free flight from November 4 – 12, when it was returned to the shuttle cargo bay. A photograph of the satellite being released is shown in Figure 7.1. For reference, the locations of the four antennas in the configuration shown in the photograph are shown in Figure 7.2 and the local and body-fixed reference frames are shown in Figure 7.3. During its mission, the CRISTA-SPAS satellite remained within a few kilometers of the Shuttle, with the primary purpose of the mission being observation of the Earth’s middle atmosphere. Three-axis control of the spacecraft was required in order to point a telescope at an area 62.9 kilometers above the WGS-84 ellipsoid. A star tracker-gyro inertial reference unit (IRU) provided attitude information for closed loop control and an Alcatel GPS receiver provided position information [Brock et al, 1995].
Figure 7.1: Photograph of CRISTA-SPAS (Courtesy of J. Rodden, Loral Space Systems).
7.3 Baseline and Line Bias Calibration

Before the residual differential phase values can be computed for the CRISTA-SPAS data, the baselines and line biases are calibrated using the batch filter described in Chapter 4.
with the entire 32 hours of data. The tracks for the line of sight vectors for all 32 hours of data is shown in Figure 7.4. Table 7.1 contains the initial estimates for the baselines in the mechanical body-fixed frame, based on the CRISTA-SPAS mechanical drawings [Ward, 1996]. The batch filter is applied using the entire set of flight data and the IRU reference quaternions, excluding any phase residuals greater than a quarter of a cycle in order to avoid including anomalous measurements in the calibration. The baselines and line biases are calibrated and are expressed in the IRU body-fixed frame, as shown in Table 7.2.

Figure 7.4: Satellite data points for the 32 hours of CRISTA-SPAS data. The data points are shown as a function of azimuth and zenith angle in the satellite local frame.
Table 7.1: CRISTA-SPAS baselines from Mechanical Drawings

<table>
<thead>
<tr>
<th>baseline</th>
<th>( x ) (meters)</th>
<th>( y ) (meters)</th>
<th>( z ) (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0325</td>
<td>0.7498</td>
<td>0.6620</td>
</tr>
<tr>
<td>2</td>
<td>0.0060</td>
<td>0.3265</td>
<td>1.2350</td>
</tr>
<tr>
<td>3</td>
<td>-0.0325</td>
<td>-0.4233</td>
<td>0.6620</td>
</tr>
</tbody>
</table>

Table 7.2: CRISTA-SPAS baselines and line biases from batch filter

<table>
<thead>
<tr>
<th>baseline</th>
<th>( x ) (meters)</th>
<th>( y ) (meters)</th>
<th>( z ) (meters)</th>
<th>line bias (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0198</td>
<td>0.7525</td>
<td>0.6635</td>
<td>0.1399</td>
</tr>
<tr>
<td>2</td>
<td>0.0175</td>
<td>0.3334</td>
<td>1.2423</td>
<td>0.0114</td>
</tr>
<tr>
<td>3</td>
<td>-0.0237</td>
<td>-0.4170</td>
<td>0.6680</td>
<td>0.2584</td>
</tr>
</tbody>
</table>

Note that the estimates of the baseline components differ from the nominal values by as much as 1 centimeter, which corresponds to an attitude error of 10 milliradians, or approximately 0.57 degrees. The discrepancy between the baseline values in the two frames could be due to a shift in the electrical phase center of the antenna after the satellite was deployed or a slight misalignment between the mechanical and IRU body-fixed frames. Using the baseline and line bias values shown in Table 7.2 and the IRU reference quaternions, the ideal differential phase residuals, containing errors due only to multipath, variations in the baselines and line biases over time, and measurement noise, can be computed for multipath modeling analysis.

7.4 Spatial Repeatability of the CRISTA-SPAS Data

Before the CRISTA-SPAS data are processed using the reflector identification method, the spatial repeatability of the data is tested. This is done by identifying GPS satellite tracks that follow the same path. In this testing, four such paths are identified. Because each of these paths pass through the same region of the sky, they should view similar
multipath trends in both the SNR and the carrier phase. The azimuth and zenith angles of the
four tracks chosen for evaluation are shown in Figure 7.5. The SNR and residual differential
phase for these satellite passes are shown in Figure 7.6 and Figure 7.7, respectively.

Figure 7.5: Four satellite passes following similar tracks, displayed in the
CRISTA-SPAS body-fixed reference frame.
Figure 7.6: SNR for the slave 3 antenna for the four satellite passes. The four colors represent four satellite passes occurring at the same spatial location but at different times.

Figure 7.7: Residual differential phase for baseline one for the four passes. The four colors represent four satellite passes occurring at the same spatial location but at different times.
Since both the SNR and residual phase follow the same trends for the similar satellite passes, it can be inferred that the data are highly spatially correlated. It is reasonable, therefore, to assume that the reflector identification method will be effective in isolating an effective reflector location that will reduce the phase residuals for not only a single pass, but other passes that follow the same path in the sky. Also, once an effective reflector has been found for that portion of the sky, there is no need to re-solve for a new location. The previous computed residuals may simply be applied or the reflector location may be iterated upon by using the previous reflector location as the a priori state.

7.5 Residual Phase Map Multipath Correction

Another means by which to test the repeatability of the carrier phase multipath in the CRISTA-SPAS spacecraft is conducted by implementing a phase map correction. A detailed explanation of the process as applied to the data from CRISTA-SPAS and the results are shown in [Axelrad and Reichert, 1997]. This involves computing phase residuals for all 32 hours of satellite data and generating a phase correction map as a function of the line of sight vector. This is based on the method presented in [Cohen and Parkinson, 1991] and [Cohen, 1992]. In the evaluation of the CRISTA-SPAS data, three methods are used to implement a fit to the data, including a spherical harmonic fit, a polynomial fit, and a correction grid method. A description of these methods is discussed in Chapter 4.

The correction maps using a one degree by 1 degree grid fit, a twelfth order spherical harmonic fit, and a sixth order two dimensional polynomial fit are shown in Figures 7.8, 7.9, and 7.10, respectively. Note that these maps are plotted as a function of the rectangular values, x and y, as shown in equation (4.9). The grid fit is termed a 1 degree by 1 degree fit because the bin size for the fit corresponds to a \( \Delta x \) and \( \Delta y \) size each of 1 degree.
Additionally, both a 2 degree and a 4 degree bin size case and an eighth order spherical harmonic fit are computed.

![1 Degree Gridded Residual Phase (mm) for Baseline 1](image)

Figure 7.8: One degree grid fit for baseline 1. The contour map displays residual differential phase errors as a function of the rectangular $x$-$y$ coordinate location.
Figure 7.9: Twelfth order spherical harmonic fit for baseline 1. The contour map displays residual differential phase errors as a function of the rectangular $x$-$y$ coordinate location.
Figure 7.10: Sixth order 2-D polynomial fit for baseline 1. The contour map displays residual differential phase errors as a function of the rectangular x-y coordinate location.

In all three of the maps for baseline 1 shown above, there exists a region of severe multipath in the lower right portion of the map. The errors range from approximately -4 to +4 centimeters and exist in an area from approximately 0 to 45 degrees in elevation. Similar, but not quite as large, phase errors are also seen in the other two baselines in all of the other maps for the same region of the sky.

Table 7.3 shows the RMS of the residual phase improvements using each of the three methods. Note that the raw residuals are smallest for baseline 3 with a residual RMS of 7.8 millimeters and largest for baseline 1 with an RMS of 9.6 millimeters. The 1 degree by 1
degree correction grid fit is shown to provide the best fit to the phase residuals by reducing the residuals to the range of 4.2 to 4.6 millimeters. The large reduction in the residual phase indicates that in fact, 40 to 50 percent of the differential phase error is spatially correlated. The remaining 4 to 5 millimeters of error is like due to high frequency phase noise and perhaps a smaller low frequency error is due to baseline and line bias variation.

An attitude point solution algorithm, developed by Ward [1996], is implemented to compute new attitude estimates using the improved phase measurements. The statistics of the resulting attitude improvements for all of the phase maps are shown in Table 7.4. The largest attitude improvements are produced using the phase corrections from the 1 degree by 1 degree grid map. After applying the grid map to the entire set of phase data, the RMS of the attitude error reduces from 0.25 degrees to 0.1 degrees in yaw, 0.14 degrees to 0.07 degrees in roll, and 0.24 degrees to 0.18 degrees in pitch. The attitude values before and after the grid map correction are shown as a function of time in Figure 7.11 and Figure 7.12, respectively.

The results for the sky maps for the eighth and twelfth order spherical harmonic fits to the residual phase data are presented in Table 7.3 and Table 7.4. The eighth order spherical harmonic phase map is shown in Figure 7.9. While Figure 7.9 shows the same general trend as the grid map, it cannot capture the irregularities in the residuals shown in the grid map. Spherical harmonics assume a level of symmetry and smoothness for low order models which do not appear to be valid for the CRISTA-SPAS reflector configuration. Furthermore, in a typical spacecraft environment, reflectors are likely to be fairly small producing irregular multipath contributions. A spherical harmonic model is perhaps more appropriate for ground applications where there are large surfaces dominating the multipath environment.

Finally, the sky map for the polynomial fit is shown in Figure 7.10 and the residual and attitude improvements are shown in the tables below. As seen in Figure 7.10, the polynomial fit is capable of representing steep residual gradients near the edges of the map,
where the edges correspond to lower elevation angles. However, false corrections are introduced for high elevation angle satellites, resulting in an overall point solution accuracy similar to the spherical harmonic models.
Table 7.3: Summary of phase residual statistics for sky map correction algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>RMS Residuals in millimeters (% Reduction compared to raw)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline 1</td>
</tr>
<tr>
<td>Raw Residuals</td>
<td>9.6</td>
</tr>
<tr>
<td>Grid Sky-Map</td>
<td></td>
</tr>
<tr>
<td>1 degree × 1 degree</td>
<td>4.7 (51%)</td>
</tr>
<tr>
<td>2 degree × 2 degree</td>
<td>5.0 (48%)</td>
</tr>
<tr>
<td>4 degree × 4 degree</td>
<td>5.6 (42%)</td>
</tr>
<tr>
<td>Spherical Harmonic Fit to Raw Data</td>
<td></td>
</tr>
<tr>
<td>8th Order</td>
<td>6.9 (28%)</td>
</tr>
<tr>
<td>12th Order</td>
<td>6.3 (35%)</td>
</tr>
<tr>
<td>Polynomial Fit to Raw Data</td>
<td></td>
</tr>
<tr>
<td>6th Order</td>
<td>6.9 (28%)</td>
</tr>
</tbody>
</table>

Table 7.4: Summary of point solution statistics for sky map algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Point Solution RMS Error in degrees (% Reduction compared to uncorrected.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yaw</td>
</tr>
<tr>
<td>Point Solution Errors w/o corrections</td>
<td>0.250</td>
</tr>
<tr>
<td>Grid Sky-Map</td>
<td></td>
</tr>
<tr>
<td>1 degree × 1 degree</td>
<td>0.102(59%)</td>
</tr>
<tr>
<td>2 degree × 2 degree</td>
<td>0.107(57%)</td>
</tr>
<tr>
<td>4 degree × 4 degree</td>
<td>0.125(50%)</td>
</tr>
<tr>
<td>Spherical Harmonic Fit to Raw Data</td>
<td></td>
</tr>
<tr>
<td>8th Order</td>
<td>0.172(31%)</td>
</tr>
<tr>
<td>12th Order</td>
<td>0.152(40%)</td>
</tr>
<tr>
<td>Polynomial Fit to Raw Data</td>
<td></td>
</tr>
<tr>
<td>6th Order</td>
<td>0.172(31%)</td>
</tr>
</tbody>
</table>
Figure 7.11: Uncorrected attitude solutions. The IRU reference attitude is represented by the dark line while the attitude point solution, uncorrected for multipath, is shown as the lighter line. The yaw, roll, and pitch angles are shown in the top, middle, and bottom figures, respectively.
Figure 7.12: Attitude solution after phase correction using the 1 degree grid map. The IRU reference attitude is represented by the dark line while the attitude point solution, corrected for multipath using the 1 degree grid map, is shown as the lighter line. The yaw, roll, and pitch angles are shown in the top, middle, and bottom figures, respectively.
This method of computing corrections to the phase using the attitude solution is somewhat flawed because it requires ideal attitude, baseline, and line bias solutions in order to accurately determine the phase residuals. Any error in the attitude, baselines, or line biases directly maps into the computation for the residual phase. Therefore it is difficult to separate attitude, baseline, and line bias errors from multipath. In order to avoid the problem, a dependence on SNR, rather than phase, is utilized because the SNR is much less sensitive to attitude errors.

7.6 Temporal SNR-Based Mitigation Method

The temporal SNR-based mitigation method was first demonstrated by Comp [1996] for a number of ground experiments and on some satellite passes for the CRISTA-SPAS spacecraft. In this research, the procedure has been automated in order to permit correction of all of the satellite passes on CRISTA-SPAS for which there are sufficient observations to perform the technique. The adaptive technique currently used takes approximately 50 to 100 observations to converge and is limited in frequency resolution to $5 \times 10^4$ Hertz over a twenty second sampling interval.

A summary of the residual phase reduction for the entire set of CRISTA-SPAS data is shown in Table 7.5. The attitude point solution is recomputed using the updated phase measurements. The point solution results for the entire 32 hours of data are shown in Table 7.6. Figure 7.13 displays the attitude point solution for a 4 hour period using the improved phase measurements.
Table 7.5: Residual phase improvement for the temporal SNR-based mitigation method.

<table>
<thead>
<tr>
<th></th>
<th>RMS of $\delta \hat{\phi}$ (mm) (%) Reduction compared to raw</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Baseline 1</td>
</tr>
<tr>
<td>Raw Residuals</td>
<td>9.6</td>
</tr>
<tr>
<td>Temporal SNR Method</td>
<td>7.2 (25%)</td>
</tr>
</tbody>
</table>

Table 7.6: Attitude point solution results for the temporal SNR-based mitigation method.

<table>
<thead>
<tr>
<th></th>
<th>RMS of Point Solution Error (degrees) (%) Reduction compared to uncorrected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yaw</td>
</tr>
<tr>
<td>Point Solution Errors without</td>
<td>0.250</td>
</tr>
<tr>
<td>Corrections</td>
<td></td>
</tr>
<tr>
<td>Temporal SNR Method</td>
<td>0.197 (21%)</td>
</tr>
</tbody>
</table>
Figure 7.13: Attitude after phase correction using the temporal SNR-based algorithm. The IRU reference attitude is represented by the dark line while the attitude point solution, corrected for multipath using the temporal SNR-based algorithm, is shown as the lighter line. The yaw, roll, and pitch angles are shown in the top, middle, and bottom figures, respectively.
Although the attitude improvement is significant—approximately 20 percent in the yaw and pitch—the results shown do not indicate as large of an improvement as the attitude solutions using the grid correction methods. One factor that inherently limits the performance of the temporal SNR-based mitigation method is the long convergence time required. Because of this, corrections are applied only to satellite arcs for which there are 200 or more data points, thus eliminating approximately twenty percent of the total data points from processing. Phase residuals for which there are no SNR-based corrections are included in their raw uncorrected form in the overall statistics. Furthermore, because the data arcs which are neglected are the short ones that tend to correspond to satellites at low elevations, they are more likely to suffer from large multipath errors, thus contributing more to the errors shown in Table 7.5, Table 7.6, and Figure 7.13.

7.7 Application of the Reflector Identification Algorithm

Utilizing the SNR, the reflector identification algorithm is implemented for the CRISTA-SPAS data. For these cases, the antenna gain values are initially computed using the Trimble antenna pattern obtained from R. Allshouse and shown in Figure 2.3 [Comp, 1996]. However, this gain pattern for the Trimble patch antennas did not match well with the received SNR values. To illustrate the mis-match between the original gain pattern and the true SNR data, the SNR and the antenna gain are shown for a single satellite pass in Figure 7.14. The antenna gain is found by multiplying the normalized gain for the Trimble patch antenna by a constant amplitude, determined using a least squares fit to the SNR data. The figure shows a large dip in the gain, which is not seen in the corresponding SNR data. For this reason, a revised antenna gain, $A_e$, is computed using a simple second order polynomial fit to the SNR data and is used instead of the Trimble gain pattern.
Figure 7.14: Antenna gain and SNR. The measured SNR, shown as the light line, is displayed with the corresponding antenna gain (computed from the Trimble pattern shown in Figure 2.3), shown as the dark line. This figure illustrates the mis-match between the antenna gain and the SNR for the CRISTA-SPAS data.

The reflector identification method is applied to seven satellite passes that occur during a half hour period (11.7 - 12.2 hours) of the CRISTA-SPAS data set. The satellites move through different parts of the sky, covering various regions of the sky map shown in Figure 7.8. If the approach is to be successful in improving attitude performance, phase data from the majority of satellites contributing to the attitude solution must be corrected. Thus, the improvement to these seven passes is evaluated as representative of the general performance. For the seven satellite passes that are considered, only PRN 1 and 25 produced favorable results. The results for these two satellite passes are discussed below.

The Nelder-Mead algorithm is implemented for reflector estimation in addition to the batch least squares algorithm. Two models are used in the batch least squares
implementation – one with the effects of phase shift upon reflection set to 180 degrees and one where only the geometrical path delay is included in the multipath relative phase term. Out of the three scenarios processed, the least squares batch scheme with only the geometrical path delay effects included produced the best results. These are the results that are presented below.

The SNR fits for the four antennas for a single pass of satellite PRN 1 are shown in Figure 7.15 through 7.18. One successful pass producing improvements in the residual phase is for the off-baseline formed by the number 2 and 3 slave antennas. The phase correction for this off-baseline is shown in Figure 7.19. Unfortunately, an appropriate effective reflector is not located for the master antenna, as is represented by the phase residual for baseline 1 shown in Figure 7.20, so the phase measurements could not be adequately updated for this satellite pass. In addition, favorable results are also found for satellite PRN 25. The state vectors and residual phase improvements for both of PRN1 and PRN 25 are shown in Table 7.7 and Table 7.8.
Figure 7.15: Measured and computed SNR fit for the master antenna, satellite 1. The gray line is the computed SNR and the black line is the measured SNR.

Figure 7.16: Measured and computed SNR fit for the slave 1 antenna, satellite 1. The gray line is the computed SNR and the black line is the measured SNR.
Figure 7.17: Measured and computed SNR fit for the slave 2 antenna, satellite 1. The gray line is the computed SNR and the black line is the measured SNR.

Figure 7.18: Measured and computed SNR fit for the slave 3 antenna, satellite 1. The gray line is the computed SNR and the black line is the measured SNR.
Figure 7.19: Residual phase fit for the slave 2-slave 3 off-baseline, Satellite 1. The gray line is the computed residual differential phase and the black line is the measured residual differential phase.

Figure 7.20: Residual phase fit for baseline 1, satellite 1. The gray line is the computed residual differential phase and the black line is the measured residual differential phase.
Table 7.7: Reflector locations and amplitudes for CRISTA-SPAS.

<table>
<thead>
<tr>
<th>PRN</th>
<th>antenna</th>
<th>( d ) (meters)</th>
<th>( az ) (degrees)</th>
<th>( el ) (degrees)</th>
<th>( A_m ) (AMU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>master</td>
<td>0.0871</td>
<td>190</td>
<td>66</td>
<td>2.34</td>
</tr>
<tr>
<td>1</td>
<td>slave 1</td>
<td>0.2046</td>
<td>190</td>
<td>64</td>
<td>2.34</td>
</tr>
<tr>
<td>1</td>
<td>slave 2</td>
<td>0.0779</td>
<td>250</td>
<td>73</td>
<td>3.92</td>
</tr>
<tr>
<td>1</td>
<td>slave 3</td>
<td>0.1907</td>
<td>220</td>
<td>73</td>
<td>4.07</td>
</tr>
<tr>
<td>25</td>
<td>master</td>
<td>0.4316</td>
<td>217</td>
<td>79</td>
<td>0.69</td>
</tr>
<tr>
<td>25</td>
<td>slave 1</td>
<td>0.3934</td>
<td>282</td>
<td>73</td>
<td>1.89</td>
</tr>
<tr>
<td>25</td>
<td>slave 2</td>
<td>0.1534</td>
<td>68</td>
<td>63</td>
<td>1.24</td>
</tr>
<tr>
<td>25</td>
<td>slave 3</td>
<td>0.1751</td>
<td>307</td>
<td>64</td>
<td>2.18</td>
</tr>
</tbody>
</table>

Table 7.8: Phase improvements for CRISTA-SPAS residual differential phase data.

<table>
<thead>
<tr>
<th>PRN</th>
<th>baseline</th>
<th>residual before (millimeters)</th>
<th>residual after (millimeters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2-3</td>
<td>5.34</td>
<td>4.28</td>
</tr>
<tr>
<td>1</td>
<td>m-2</td>
<td>7.51</td>
<td>7.36</td>
</tr>
<tr>
<td>25</td>
<td>m-1</td>
<td>6.41</td>
<td>5.56</td>
</tr>
<tr>
<td>25</td>
<td>m-2</td>
<td>6.79</td>
<td>6.27</td>
</tr>
</tbody>
</table>

Tables 7.7 and 7.8 list results for the baselines in which the phase residuals are improved: PRN 1, baseline 2 and PRN 25, baselines 1 and 2. In addition, the phase improvement for the off-baseline formed by the slave number 2 and 3 antennas is also shown. Table 7.7 includes the reflector location and the multipath amplitude and Table 7.8 shows the residual differential phase before and after the phase corrections are applied. For the satellite/baseline combinations in which phase residuals are improved, the phase measurements are corrected and an attitude point solution is recomputed using the algorithm developed by Ward [1996]. Because only a total of three baselines are corrected with minimal phase improvements, no appreciable attitude improvement is found.

The results found by implementing the method for the CRISTA-SPAS flight data indicate some potential to identify a reflector capable of reducing carrier phase multipath.
However, these results demonstrated that the ability to identify a reflector from SNR data is limited by the poor knowledge of the antenna gain pattern and antenna near field effects.
CHAPTER 8

CONCLUSIONS AND FUTURE WORK

8.1 Summary

Methods are developed in this dissertation for reducing carrier phase multipath in a post-processing manner, utilizing the signal to noise ratio and the spatial correlation found in multipath. The reflector identification algorithm utilizes the fact that both the carrier phase and the SNR are a function of the multipath relative phase, which is solely dependent on the location and orientation of a reflector. The theoretical development of the multipath relative phase is based on two contributions: the physical path delay of the signal and the change in phase of a circularly polarized signal upon reflection. Both of the effects contributing to multipath relative phase are derived in detail as a function of the location and orientation of the reflector.

In addition to the reflector identification algorithm, phase-based correction maps are computed using the residual phase data from the CRISTA-SPAS spacecraft. The correction maps are computed using three types of fits: a spherical harmonic fit, a two dimensional polynomial fit, and a grid fit.

A baseline and line bias calibration scheme is developed in order to produce accurate differential phase residuals. The calibration scheme makes use of all available phase data, compiled in a batch, in order to produce the best possible line bias and baseline estimates. The improved baselines and line biases are used in order to compute accurate differential
phase residuals for production of the residual phase sky maps and for accurate comparison to computed phase corrections.

8.2 Conclusions

For the CRISTA-SPAS data, the phase-based sky map correction method works very well in reducing the phase residuals and improving the attitude solution. Most notably, the 1 degree by 1 degree grid map produced the best results for both the residuals and the attitude. A downfall of this method is that it requires accurate knowledge of the attitude, baseline, and line bias solution in order to have reliable phase residuals for computing a map. Any error in these quantities directly maps into the phase residual and causes the map to be distorted.

In an attempt to avoid the previous problem, the reflector identification algorithm is implemented. The algorithm is initially tested using the NEC-BSC simulation software and produces phase improvements ranging from 83 to 94 percent. The source of error for the simulations is caused by the varying multipath amplitude, which is modeled as a constant value in the reflector identification algorithm.

The algorithm is also shown to work reasonably well for ground data, improving the residuals by as much as 22 percent. In cases where no significant residual phase improvement is seen, the correction profile from the algorithm does seem to closely follow the trends of the true multipath, indicating some low frequency error remains. The low frequency error could be due to an error in the baselines or line biases or it could be due to an additional reflector causing multipath. However, the results for the ground data are exceptionally good considering that only very short data arcs are utilized in order to identify a reflector location. With longer data sets, it is likely that even more improvement would be seen.
Unfortunately, the algorithm does not produce significant improvement in the CRISTA-SPAS flight data. A number of possible causes for the poor performance exist, including phase pattern variations causing errors unrelated to multipath, large variations in the multipath amplitude not accurately modeled by a constant value, a lack of knowledge of the antenna gain pattern, or inappropriate application of the far field assumption. The last two possibilities are very likely because the antenna gain pattern is shown to deviate significantly from the measured SNR data and all of the reflectors identified for CRISTA-SPAS lie near the boundary between the far and near field. Because of the limited sizes of spacecraft, these results may indicate that the reflector identification algorithm is better suited for applications where the reflection surfaces are not so localized about an antenna. This points to a difficulty for using this method for satellite attitude applications.

8.3 Future Work

The analysis described here provides an important starting point for several future extensions and new applications. It would be useful to investigate using both the SNR and the pseudorange as observables of multipath. Because both display multipath in phase with one another, the two observables could be used in conjunction to identify multipath in the signal. An additional advantage to using the pseudorange as an observable is that its use is not dependent on accurate knowledge of the antenna gain pattern. Also, the research in this dissertation can be extended by composing a batch or sequential filter for GPS position and velocity estimation with the multipath parameters of amplitude and phase included in the state matrix.

Other future work is to test the applicability of this reflector identification method to reference sites for IGS and kinematic GPS applications. For these types of applications, it
would also be useful to investigate the ability to identify and correct very low frequency multipath errors, due to reflectors close to the antenna.

Finally, an interesting new application is to investigate multipath environment for spacecraft applications. In this situation, based on the conclusions shown in this dissertation, modeling of spacecraft contributions to the antenna pattern would be very useful. Specifically, this would include investigation of the near field effects and the change of the antenna pattern due to the near field reflective surfaces.
BIBLIOGRAPHY


