BISTATIC SCATTERING
OF GLOBAL POSITIONING SYSTEM SIGNALS
FROM ARCTIC SEA ICE

by

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The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline.
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Bistatic scattering of Global Positioning System signals from Arctic sea ice
Thesis directed by Professor Penina Axelrad
and Associate Research Professor James Maslanik

Abstract - This work evaluates the potential use of scattered Global Positioning System (GPS) signals for the retrieval of information related to the presence and condition of the Arctic sea ice cover.

Although the combined satellite microwave record over the period 1979 to 2006 indicates that the Arctic sea ice extent has declined for every month, evidence for accompanying reductions in ice thickness has been hampered by the poor separability of the sea ice categories based on available active and passive measurements.

Using data collected from an airborne platform flying over the Beaufort, Bering and Chukchi Seas during the month of March in 2003, a geophysical model function is fit to scattered GPS waveforms to estimate parameters such as the L-band dielectric permittivity of the surface and its large scale roughness. These products are compared satisfactorily against independent measurements of roughness provided by a LIDAR surface profiler and a reference classification of sea ice types obtained from the simultaneous analysis of polarimetric brightness temperatures at 11, 19 and 37 GHz, C-band radar backscatter and visible/infrared imagery. We find that the ability of GPS derived permittivities to delineate the extent of the sea ice cover and separate the thin ice classes matches that of the currently operational radiometers, and that GPS derived roughness measurements provide improved separability of the thick ice classes, along with additional information about the deformation processes that affect the Arctic cover.
“Cognition is in our eyes a thing of beauty and worth, and this is true of one cognition more than another, either because it is exact or because it relates to more important and remarkable objects”

*De Anima, Aristotle*
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<th>Definition</th>
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<tr>
<td>A</td>
<td>Scattering area</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Carrier wavelength</td>
</tr>
<tr>
<td>$h$</td>
<td>Receiver altitude</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Satellite elevation</td>
</tr>
<tr>
<td>$T_{CA}$</td>
<td>GPS C/A code chip length</td>
</tr>
<tr>
<td>$T_{int}$</td>
<td>Integration (detection) time</td>
</tr>
<tr>
<td>$r_{CA}$</td>
<td>Radius of first chip zone</td>
</tr>
<tr>
<td>$\varepsilon = \varepsilon' + i \varepsilon''$</td>
<td>Dielectric permittivity</td>
</tr>
<tr>
<td>$\Re$</td>
<td>Fresnel reflection coefficients</td>
</tr>
<tr>
<td>$\delta_p$</td>
<td>Penetration depth</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Signal delay</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Carrier angular frequency</td>
</tr>
<tr>
<td>$\Delta \omega$</td>
<td>Doppler shift in frequency</td>
</tr>
<tr>
<td>$\Lambda_{CA}(\tau)$</td>
<td>GPS C/A code autocorrelation function</td>
</tr>
<tr>
<td>$\chi(\tau, \Delta \omega)$</td>
<td>GPS ambiguity function</td>
</tr>
<tr>
<td>$\sigma^0$</td>
<td>Normalized scattering cross section</td>
</tr>
<tr>
<td>$P_r(\tau)$</td>
<td>Scattered power</td>
</tr>
<tr>
<td>$mss$</td>
<td>Mean square slope (one-dimensional)</td>
</tr>
<tr>
<td>$\tan \beta$</td>
<td>Root mean square slope</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Solid angle</td>
</tr>
<tr>
<td>$r={x,y,0}$</td>
<td>Surface position vector</td>
</tr>
<tr>
<td>$T$</td>
<td>Physical temperature</td>
</tr>
<tr>
<td>$T_B$</td>
<td>Radiometric (brightness) temperature</td>
</tr>
<tr>
<td>$\epsilon(\theta)$</td>
<td>Surface emissivity</td>
</tr>
<tr>
<td>$\tau_p(\theta)$</td>
<td>Hemispherical directional transmittance</td>
</tr>
<tr>
<td>$\xi(x,y)$</td>
<td>Surface height (random function)</td>
</tr>
<tr>
<td>$\text{PDF}_h$</td>
<td>Probability density function of surface heights</td>
</tr>
<tr>
<td>$\text{PDF}_{\nabla h}$</td>
<td>Probability density function of surface slopes</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Surface root mean square height</td>
</tr>
<tr>
<td>$L$</td>
<td>Surface correlation length</td>
</tr>
<tr>
<td>$\rho(r,r')$</td>
<td>Autocorrelation function of surface heights</td>
</tr>
<tr>
<td>$S(k)$</td>
<td>Power spectral density (spectrum) of roughness</td>
</tr>
</tbody>
</table>
$X_i(q) –$ Characteristic function of the random variable $h$

$p_0, p –$ Input and output polarization vectors

$n_{inc} –$ Unit vector along direction of incidence

$n_{scat} –$ Unit vector along direction of scatter

$n –$ Surface normal vector

$F(\hat{n}_{inc}, \hat{n}_{scat}) –$ Scattering amplitude

$\tilde{E}(\hat{n}_{inc}, \hat{n}_{scat}) –$ Fourier amplitude

$\kappa_{scat} = \{\kappa, \kappa_z\} –$ Scattered wave vector

$k_{inc} = \{k_\perp, k_z\} –$ Incident wave vector

$k = \omega \sqrt{\varepsilon_0 \mu_0} –$ Vacuum wave number

$q = \kappa_{scat} - k_{inc} = \{q_\perp, q_z\} –$ Scattering vector
LIST of ACRONYMS

ACF – AutoCorrelation Function
AGC – Automatic Gain Control
ATM – Airborne Topographic Mapper
C/A – Code Acquisition
FWHM – Full Width Half-Maximum
FYI – First Year Ice
GNSS – Global Navigation Satellite System
GPS – Global Positioning System
GR – Spectral Gradient Ratio
IGS – International Geodetic Service
JPL – Jet Propulsion Laboratory
KA – Kirchhoff Approximation
KAGO – Kirchhoff Approximation in Geometric Optics
LHCP – Left Hand Circularly Polarized
LIDAR – LIght Detection And Ranging
LOS – Line Of Sight
MKS – Meter Kilogram Second
MODIS – MODeRate resolution Imager Spectrometer
MSS – Mean Square Slope
MYI – MultiYear Ice
NASA – National Aeronautics and Space Administration
NI – New Ice
NIC – National Ice Center
NOAA – National Oceanic and Atmospheric Administration
OW – Open water
PDF – Probability Density Function
PR – Polarization Ratio
PRN – Pseudo Random Noise
PSD – Power Spectral Density
PSR – Polarimetric Scanning Radiometer
RGB – Red Blue Green
RHCP – Right Handed Circularly Polarized
RMS – Root Mean Square
SAR – Synthetic Aperture Radar
SNR – Signal to Noise Ratio
SPM – Small Perturbation Method
SSA – Small Slope Approximation
UTM – Universal Transverse Mercator
WGS – World Geodetic System
WMO – World Meteorological Organization
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0 – Introduction

0.1 Introduction

Of all the recent changes in the environment, the gradual loss of the Arctic sea ice cover stands out prominently ([Serreze et al., 2003], [Stroeve et al., 2005]). The floating sea ice cover in the polar oceans has been routinely monitored by satellite sensors from approximately 1978 to the present, providing a wide base of knowledge about microwave and optical signatures of sea ice that makes it possible to track the global extent of sea ice and to classify it into broad categories of age/thickness. It is the aim of this thesis is to evaluate the potential of a novel observation technique, the bistatic scattering of Global Positioning System (GPS) signals, for augmenting the existing observation techniques in the retrieval of sea ice parameters.

The analysis of reflected L-band GPS signals for remote sensing made its first appearance in 1993 [Martin-Neira, 1993], motivated by the availability of the present GPS, GLONASS and the future GALILEO constellations of navigation satellites. Initially conceived as a means to perform radar altimetry over the oceans, reflected GPS signals have proven able to measure sea level with centimeter precision ([Treuhaft et al., 2001], [Lowe et al., 2002]) and be responsive to ocean winds speeds ([Cardellach, 2001], [Garrison et al., 2002], [Katzberg et al., 2006]), soil moisture changes ([Masters, 2004], [Katzberg et al., 2005]) and the presence of sea ice [Komjathy et al., 2000]. The utilization of the bistatic radar technique requires only a slightly modified GPS receiver to register the scattered signals, and the application of a radar cross-section model that represents the signal-surface interaction for the eventual inversion of geophysical parameters ([Hajj & Zuffada, 2003], [Zavorotny & Voronovich, 2000]).
The first step towards the construction of a radar cross-section model involves identifying the primary scattering mechanisms at work in different scenarios. The sea ice cover is a statistically uneven layer of saline ice floating on sea water under a layer of snow. The wavelength of the GPS signal is about 100 times larger than the size of the dielectric inhomogeneities typically found in sea ice (i.e. snow grains, air and brine pockets), indicating that effects of volume scattering be excluded and surface scattering be treated as the only mechanism contributing to the L-band radar cross-section. A good portion of our work has dealt with the compilation of analytical models that describe the interaction of electromagnetic radiation with statistically rough surfaces, providing the theoretical basis and the forward model that facilitates the retrieval of sea ice parameters from scattered GPS waveforms.

An airborne experiment was conducted over Arctic sea ice to test and evaluate the performance of the GPS bistatic radar technique for the mapping of sea ice conditions using different forward models. The procedure for model inversion, which is based on a least squares minimization of the model-to-waveform residuals in a multi-dimensional space of model parameters, yields surface reflectivity, roughness and topography. The inversion products from the GPS bistatic radar are compared against a set of co-located data that include LIDAR terrain profiles and roughness spectra, multichannel microwave radiometer and synthetic aperture radar (SAR) imagery.

0.2 Motivation

The significance of this study is twofold. First, we build on previous work on the theoretical modeling of electromagnetic scatter from sea ice, which remains key to the interpretation of current microwave radiometer and radar observations. The rich variety of bistatic transmitter-receiver configurations using GPS signals, which provides viewing geometries that range from normal incidence to low grazing angles, makes the study of
scattered GPS waveforms an ideal means for the exhaustive validation of a theoretical model of the surface scattering cross-section. Second, the proposed GPS bistatic radar measures the forward scatter of L-band signals from sea ice, which is a probing mode not presently available from any other spaceborne remote sensing system. We evaluate the capability of this novel GPS radar system to complement the information provided by other polar sensors, and facilitate an assessment of its operational requirements and limitations. The main contributions of this thesis are summarized as follows:

1) The formulation of a quantitative link between sea ice surface roughness and sea ice thickness categories, which allows the former to be used as a proxy for the latter.

2) The development of a theoretical framework for the quantitative inversion of sea ice parameters from measurements of electromagnetic surface scatter at L-Band, assisted and validated with experimental data.

3) The evaluation of the operational requirements and limitations of a GPS bistatic radar system for the observation of sea ice conditions.

0.3 Overview

The analysis of scattered GPS signals from sea ice draws from various areas of knowledge. The first part of this thesis provides the background required for understanding the structure of the probing GPS signal, the concept and peculiarities of the bistatic radar geometry and the characteristics of the sea ice cover that account for the mechanisms by which it interacts with the probing signal.

A critical survey of the observation techniques currently used to monitor sea ice sets the stage in which the analysis of the GPS bistatic radar takes place. Scattered GPS signals can be modeled as a superposition of secondary waves arising from the sea ice surface
with different delays and weighted in power by the surface scattering cross-section, $\sigma^0$.

The second part of this thesis delves into the theoretical modeling of the scattering cross-section of a statistically rough surface. We start by examining the physical principles that underlie the construction of the so-called classical models (i.e. the Kirchhoff Approximation and the Small Perturbation Method) along with their domains of validity, and then follow on to present a more robust approach (i.e. the Small Slope Approximation) that overcomes the limitations of the former.

The utilization of a theoretical model for the scattering cross-section of the sea ice surface facilitates the link between scattered signals and surface parameters, and thus enables the construction of a forward model that relates the shape and strength of the scattered GPS signal to the spectrum of sea ice roughness and the sea ice dielectric permittivity. The inversion of sea ice parameters from scattered GPS signals is carried out by a non-linear fitting procedure based in a least squares minimization of the model-to-measurement residuals in a multi-dimensional space of surface parameters.

The third part of this thesis deals with the experimental validation of the geophysical forward model at extracting surface parameters from scattered GPS signals. The various data sets acquired during an airborne experimental campaign over the Arctic ice pack provide an exceptional set of co-located “ground truth” measurements against which the GPS inversion products can be contrasted. LIDAR surface profiles, passive microwave brightness temperatures, SAR backscatter and visible/infrared imagery are all used to characterize the variety of sea ice conditions present in the data sets. Taking the LIDAR profiles and the radiometer/SAR/visible classification of sea ice as a reference, we verify the usefulness of GPS retrievals at determining surface roughness and providing a classification of sea ice that helps overcome the limitations that affect other observation techniques.
0.4 Thesis outline

This thesis has been structured into three parts. **Chapter I** provides the background necessary to lay down the remote sensing problem of bistatic scattering of GPS signals. The structure of the **probing GPS signal** is introduced first, with special mention to its autocorrelation properties. The geometry of observation of a bistatic radar is described next, along with the **radar equation** for a point target, which relates the power emitted by a transmitter to that received by a detector using a parameter referred to as the target cross-section. The bistatic radar equation for a point target can be generalized to an extended target such as sea ice under GPS illumination by taking into account the way that the GPS autocorrelation function effectively delineates the bistatic radar footprint, i.e. the area where effective contributions to the detected power come from. An analytical model of the scattered GPS signal arises from this treatment in terms of the as-of-yet unknown surface normalized **cross-section**, $\sigma_0$, a parameter that encodes the mechanisms by which the probing signal becomes re-radiated by the extended target. Further insight into these re-radiation mechanisms is gained by looking at the **internal structure** and **dielectric properties** of the different sea ice forms that populate the polar oceans. The last section in Chapter I is dedicated to the observation techniques currently used to monitor the state of the sea ice cover, in an attempt to understand their abilities and limitations and to set the context within which we embark the study of scattered GPS signal.

**Chapter II** undertakes the modeling of the scattering cross-section of a **statistically rough surface**. Emphasizing first principles, the construction of various approximations to the problem of electromagnetic scattering from rough surfaces is presented. All these approximations invariably arise from imposing the continuity of tangential electric and magnetic fields on a surface characterized by statistical moments of second order, such as
the autocorrelation function of roughness. For smoothly undulating surfaces, the *Kirchhoff approximation* provides the best approximation to the scattering cross-section, which is expressed in terms of the dielectric permittivity of the surface and the mean square slope (mss) of roughness. For a general kind of surface, this approximation works best for a range of angles that lie near the direction of specular (flat surface) reflection. Surfaces that have a greater content of roughness at scales smaller than the observing wavelength are best described by the *perturbation theory*, in which the scattering cross-section appears proportional to the fourth power of the observing wavenumber (like Rayleigh scattering from a volume filled with inhomogeneties) and proportional to the spectrum of roughness. This approach gives best results for angles far off the specular direction. The family of *two-scale models* tries to overcome the limitations that come along the two former (a.k.a. classical) models by postulating the existence of a certain roughness wavenumber cutoff (or limiting scattering angle) that neatly demarcates the (supposedly overlapping) domains of applicability of either classical model. We find that the existence of the two-scale wavenumber cutoff cannot be guaranteed and therefore refrain from pursuing the two-scale approach any further. An alternative solution is provided by the *small slope approximation*, which is constructed regarding some general considerations about the symmetry of the scattering problem and preserves the Kirchhoff approximation and the perturbation theory as limiting solutions.

**Chapter III** brings in the experimental part of this thesis. We begin with a brief description of the instruments that took part in the **AMSRIce03 validation campaign**: an airborne polarimetric radiometer working at 11, 19 and 37 GHz, the RADARSAT synthetic aperture radar, the MODIS visible/infrared spectrometer, a LIDAR profiler and the GPS bistatic radar. Next we introduce the Arctic scenarios that were overflown and follow directly onto the formation of a **reference classification** of sea ice types based on the clustering of passive microwave signatures, which become labeled after a contextual interpretation effort made in collaboration with SAR and MODIS images. The surface
profiles provided by the LIDAR sensor are processed into surface roughness power spectra and parameterized in terms of exponential RMS heights and correlation lengths, which provide reference parameters for the characterization of sea ice roughness. Finally, the process of parameter extraction from scattered GPS signals is carried out using both the Kirchhoff and Small Slope approximations for the scattering cross-section of the sea ice surface, to generate L-band permittivities and surface roughness products, and the results compared against the references available. Finally, the separability of the reference sea ice classes in terms of the new GPS retrievals is examined.
CHAPTER I – Background
1 Introduction

The problem of bistatic scattering of GPS signals from sea ice requires knowledge from several disparate fields. We want to evaluate the performance of a remote sensing technique that utilizes signals from the Global Navigation Satellite Systems (GNSS) to probe into a natural surface. In this part we lay out the necessary background upon which further discussion lies: from the structure of the probing GPS signal, through the specifics of the bistatic technique, to the properties of the target that facilitate its interaction with the probe.

2 The probe: GPS signals

The signals transmitted by the constellation of GPS satellites provide continuous and global L-band coverage of the Earth’s surface. Of concern to us here, the GPS signal consists of an embedded binary modulation (C/A code) on a right hand circularly polarized (RHCP) carrier at a frequency (L1) of 1.575 GHz. The modulation code is a pseudo-random noise (PRN) sequence with a repetition period of ~1 msec and a code chip frequency of $1/T_{CA} = 1.023$ MHz, which spreads the signal spectrum well under typical antenna noise levels:

$$GPS(t) = CA(t) \cos(\omega t + \phi)$$

(2.1)

![Figure I.1 – GPS L1 signal: C/A code modulation and carrier (Not to scale)](image)
The conventional observables of a GPS receiver, signal delay and Doppler shift, are determined after cross-correlation of the measured GPS signal $GPS(t)$ in (2.1) with locally generated in-phase $I(t)$ and quadrature $Q(t)$ replicas (a.k.a. matched filtering):

$$
I(t) = 2CA(t + \tau) \cos[(\omega + \Delta\omega)t]
$$

$$
Q(t) = 2CA(t + \tau) \sin[(\omega + \Delta\omega)t]
$$

The map of cross-correlations between the GPS signal and a suite of delay and Doppler shifted replicas forms the GPS autocorrelation function $\chi(\tau, \Delta\omega)$ (a.k.a. ambiguity function in radar terminology, [Skolnik, 1990]), which can be expressed as:

$$
\chi(\tau, \Delta\omega) = \left|<GPS(t)I(t)>\right|^2 + \left|<GPS(t)Q(t)>\right|^2 = |\Lambda_{CA}(\tau)|^2 \cdot \left|\frac{\sin(\Delta\omega T_{int}/2)}{\Delta\omega T_{int}/2}\right|^2
$$

$$
\Lambda_{CA}(\tau) = \langle CA(t)CA(t + \tau) \rangle = \begin{cases} 1 - |\tau|/T_{CA} & |\tau| < T_{CA} \\ 0 & otherwise \end{cases}
$$

Where $\tau$ and $\Delta\omega$ refer to the code delay and frequency offset of the replica relative to the incoming signal, $\Lambda_{CA}(\tau)$ is the autocorrelation function of the PRN sequence (represented by a triangle function with a width at the base equal to $2T_{CA}$), and $T_{int}$ is the coherent detection (integration) time. The GPS autocorrelation function is essentially zero for replicas that are offset by more than $T_{CA}$ in delay and more than $1/T_{int}$ in frequency relative to the incoming signal (see Figure I.2).

![Figure I.2 –GPS autocorrelation function $\chi(\tau, \Delta\omega)$ ($T_{int} = 1$ msec)](image-url)
3 The technique: Bistatic scattering

A radar system consists of a transmitting source that radiates electromagnetic energy outwards and a receiver that collects the energy that bounces back from a given target. For most radar systems, a single antenna operates as both the transmitter and the receiver (e.g. radar altimeter and synthetic aperture radar), but in a bistatic radar system, the transmitter and receiver remain at separate locations (see Figure I.3).

When a transmitted pulse encounters a smooth target-interface between different dielectric media, specular Snell’s reflection occurs and most of the return power arises from a small area about the specular point (i.e. the first Fresnel zone, of roughly the size of a radiation wavelength [Beckmann & Spizzichino, 1963]). In this case, the transmitted pulse preserves most of its structure after interacting with the target-interface, enabling its precise location relative to the receiver [Belmonte & Martin-Neira, 2006]. But if the interface is sufficiently rough, scattering effects come into play and spread the return power onto a much wider area about the specular point, termed the glistening zone. This results in a superposition of power returns arising from points on the surface, each with a distinctive Doppler frequency (a function of its location relative to the receiver velocity vector) and time delay (a function of its distance to the specular point or point of least delay). The surface area that is sensed by the scattered pulse is ultimately determined by the extent of the glistening zone, with a lower limit set by the first Fresnel zone.
3.1 The radar equation

The bistatic radar equation expresses the conversion of the power delivered by a transmitter into a measurable signal-to-noise ratio at a detector, after interaction with a target. The radar equation is simple in concept but it covers a number of assumptions about the way the radar system operates [Ulaby et al., 1986]. For a single point target, the radar equation is given by:

\[
\text{SNR} = \frac{P_t}{K T_s B} = \left( \frac{P_t G_t}{4 \pi R_1^2} \right) \sigma_b \left( \frac{1}{4 \pi R^2} \right) \left( \frac{\lambda^2}{4 \pi} \right) \left( \frac{1}{k_B T_s} \right)
\]  

(3.1.1)

Where (see Figure I.4)

- \(\text{SNR}\) = Signal-to-Noise Ratio
- \(P_t\) = Power transmitted
- \(G_t\) = Transmitter antenna gain
- \(R_1\) = Mean distance from transmitter to radar footprint on the surface
- \(R\) = Mean distance from receiver to radar footprint on surface
- \(\lambda\) = Radar wavelength
- \(G_r\) = Receiver antenna gain
- \(k_B\) = Boltzmann’s constant
- \(T_s\) = Detector temperature (scene + receiver thermal noise)
- \(B\) = Detector bandwidth
- \(\sigma_b\) = Bistatic radar cross-section

![Figure I.4 – Bistatic radar system parameters](image-url)
For a GPS conventional receiver configuration, the nominal minimum power received into an isotropic antenna \((G_r=1)\) on Earth will be given by [Navstar, 1991]:

\[
P_{\text{direct}} = \left( \frac{P G_r}{4\pi R_i^2} \right) \left( \frac{\lambda^2}{4\pi G_r} \right) \approx -160dBw
\]  

(3.1.2)

This power budget is modified by the presence of a single point target characterized by a bistatic cross-section, \(\sigma_b\), with dimensions of area, which conveys the efficiency with which the target intercepts the transmitted power and redirects it towards the receiver:

\[
P_r = P_{\text{direct}} \cdot \sigma_b \cdot \left( \frac{1}{4\pi R^2} \right)
\]  

(3.1.3)

The generalization of the radar equation for an extended target-interface observed with a cross-correlation GPS receiver is straightforward. The bistatic cross-section \(\sigma_b\) is divided by the target area to form a dimensionless (normalized) cross-section, \(\sigma^0\), which is then integrated over the pulse-limited support provided by the GPS autocorrelation function \(\chi(\tau,\Delta\omega)\) over the target-interface:

\[
P_r(\tau,\Delta\omega) = \int_{\text{surface}} \left( \frac{P G_r}{4\pi R_i^2} \right) \left( \frac{\lambda^2}{4\pi G_r} \right) \sigma^0(r) \frac{\chi[\tau - \tau'(\mathbf{r}),\Delta\omega - \Delta\omega'(\mathbf{r})]}{4\pi R^2} d^2 r
\]  

(3.1.4)

The radar equation can be simplified, assuming far field illumination and a uniform receiver gain pattern over a scattering area without significant Doppler spread, such that:

\[
P_r(\tau) = \left( \frac{P G_r}{4\pi R_i^2} \right) \left( \frac{\lambda^2 G_r}{4\pi} \right) \int_{\text{surface}} \sigma^0(\mathbf{r}) \chi[\tau - \tau'(\mathbf{r})] d^2 r
\]  

(3.1.5)

### 3.2 Pulse limited resolution

A scattered GPS waveform \(P_A(\tau,\Delta\omega)\) is the map of cross-correlations between a scattered signal [a superposition of surface returns with varying delays and dopplers, \(\tau'(x,y)\) and \(\Delta\omega'(x,y)\)] and a set of local GPS pulse replicas shifted \(\tau\) in delay and \(\Delta\omega\) in frequency.
Since, by virtue of its autocorrelation properties, a single local replica only provides support for surface returns such that $|\tau'(x,y) - \tau| < T_{CA}$ and $|\Delta\omega'(x,y) - \Delta\omega| < 2\pi/T_{int}$, the spatial resolution associated with a certain waveform sample is delimited by contours of constant delay and constant Doppler on the surface. On a plane tangent to a spherical Earth about the specular point (SP) with infinitely distant transmitter (T) and receiver (R) locations, isodelay contours $\tau'(x,y)$ are approximated by (see Figure I.5):

$$\tau'(x, y) = |RP(x, y)| + |PT(x, y)| - (R_R + R_T) = \frac{\sin \gamma}{2h} (x^2 + y^2 \sin^2 \gamma) = \text{constant}$$

Isodelay contours form a family of ellipses distributed concentrically about the specular point, with semi-major $a$ and semi-minor $b$ axes given by:

$$a = \sqrt{2\tau'h / \sin^3 \gamma} \quad b = \sqrt{2\tau'h / \sin \gamma} \quad (3.2.1)$$

Where $\tau'$ is the delay of the scattered return relative to the specular ray, $h$ is the receiver height relative to the tangent plane and $\gamma$ is the satellite elevation angle. Assuming a distant transmitter and a still surface, isodoppler contours $\Delta\omega'(x,y)$ are drawn by the
projection of the receiver velocity \((v_R)\) upon the line-of-sight unit vector (LOS) of the return, as (see Figure I.6):

\[
\Delta \omega'(x, y) = \frac{2\pi}{\lambda} v_R \cdot \cos \alpha(x, y) = \text{constant}
\]

Isodoppler contours form a family of hyperbolas with focii oriented along the projection of receiver velocity onto the tangent plane with semi-major \(a'\) and semi-minor \(b'\) axes given by:

\[
a' = h / \tan \alpha \quad b' = h
\]

Where \(\alpha\) is the angle between the receiver velocity and the LOS vector of the return.

Figure I.6 – Construction of isodoppler contours

Figure I.7 – Isodelay (blue) and isodoppler (red) contours for GPS C/A code and nadir incidence (\(r_{CA}\) is the radius of the first chip zone)
Figure I.7 above shows isodelay contours corresponding to the first five chip zones \((\tau'/T_{CA} = 0 \ldots 4)\) and isodoppler lines to \(\pm 500\) Hz for nadir incidence and airborne receiver altitudes. In order to understand how the surface cross-section is convolved with the receiver impulse response (i.e. the replica autocorrelation function) to form a scattered waveform, recall that the active scattering area of a local replica with delay \(\tau\) is delimited by points on the surface such that \(|\tau'(x,y) - \tau'| < T_{CA}\). Thus as the replica/waveform delay \(\tau\) increases, the active scattering area seeks surface points with longer delays, moving farther away from the specular point, exploring different sectors in the angular pattern of the surface cross-section.

Figure I.8 – Pulse limited resolution: the correlation output at increasing delays probes into areas further away from the specular point.
As illustrated in Figure I.8, the first scattered (specular, \( \tau' = 0 \)) returns start arriving for waveform delays \( \tau/T_{CA} \) between -1 and 0, up until the whole first chip zone becomes active, forming the *leading edge* of the waveform. The size of the active area continues to increase for increasing waveform delays, up until the whole first and second chip zones become active for \( \tau/T_{CA} = 1 \). For waveform delays \( \tau/T_{CA} \) greater than 1, the active area detaches from the specular point and becomes an expanding ring of constant area \( A_{chip} = 4\pi T_{CA} h / \sin^2 \gamma \), forming the *trailing edge* of the waveform.

Doppler spread effects will cause all surface returns shifted in frequency by more than the detection bandpass to be filtered out of the waveform. Figure I.9 below displays the typical radius of the Doppler bandpass zone (±500 Hz FWHM for \( T_{int} = 1 \) msec) versus that of the glistening zone (at 1/e level, using a quasi-specular KAGO scattering model with \( mss = 0.02 \), see Chapter II) for a receiver flying at 200 m/s. At typical airborne altitudes and speeds, the glistening zone lies totally within the detection bandpass area, indicating that Doppler spread is not an issue of concern.

\[
\begin{align*}
\text{Doppler radius (FWHM)} & = \frac{h}{\tan[\arccos(\lambda/2v_{CA}T_{int})]} \\
\text{Glistening radius (1/e)} & = h \sin \gamma \sqrt{8mss} \\
\text{Code chip radius} & = \sqrt{2h T_{CA} / \sin^2 \gamma}
\end{align*}
\]

Figure I.9 – Code correlation (first five chip zones), glistening and Doppler bandpass zones at typical airborne receiver altitudes and nadir incidence
The problem of inversion thus lies in the deconvolution of the surface cross-section \( (\sigma^0) \), itself a function of the surface properties, from the distribution of power across the waveform, \( P_r(\tau) \). The deconvolution process is simple when the glistening radius is much larger than the correlation width of the replica (~ 1/2 \( T_{CA} \) chip radius FWHM), since in that case we can assume slow variations of the cross-section within each active area and map a smoothed version of the cross-section directly into the scattered waveform as:

\[
P_r(\tau) = \left( \frac{P_G G_r}{4\pi R_0^2} \right) \left( \frac{\lambda}{4\pi R} \right)^2 A_r \left\{ \sigma^0 \right\}_r = \frac{\chi}{2\pi^2} \int_{-\infty}^{\infty} \chi(\tau - \tau'(\vec{r}))d^2\vec{r}
\]

However, if the size of the glistening zone is smaller than the correlation width of the probe (e.g. at receiver altitudes below 4000 m using the GPS C/A code, see Figure I.9, or at low signal elevations), then the deconvolution of surface properties from waveforms becomes more of an ill-conditioned problem, requiring \textit{a priori} constraints on the solution (such as an underlying model of the cross-section or other information) to facilitate the inversion (for more on inverse problem theory, see [Twomey, 1977] and [Wing, 1991]).

### 3.3 Scattered GPS waveforms

Scattered waveforms are characterized by a time delay relative to the direct signal, which translates into a ground range, a signal-to-noise level, which relates to the permittivity of the surface, and a spread in time delay, which relates to surface roughness (see Fig. I.10).

![Figure I.10 – Direct (left) and scattered (right) GPS correlation amplitudes](image-url)
The measured correlation power of the direct and scattered signals $P_{\text{direct}}$ and $P_{\text{reflected}}$ can be modeled from the radar equation as:

$$P_{\text{direct}}(\tau_d, \omega) = AGC_{\text{up}} \left( \frac{\lambda}{4\pi} \right)^2 \frac{P_G}{R_0^2} G_r^{\text{up}}(\theta, \phi) \mathcal{X}(\tau_d, \omega) \quad (3.3.1)$$

$$P_{\text{reflected}}(\tau_r, \omega) = AGC_{\text{down}} \left( \frac{\lambda}{4\pi} \right)^2 \frac{P_G}{R_0^2} G_r^{\text{down}}(\theta, \phi) \mathcal{X}_{\text{scat}}(\tau_r, \omega) \quad (3.3.2)$$

Where:

- $AGC_{\text{up}}$, $AGC_{\text{down}}$ are the receiver gain factors for the up and down-looking receiver channels. They act before the input power turns into digital counts, setting the antenna input noise voltage to optimum levels for digitization.
- $P_G R_0^2$ accounts for the combined effects of a given transmitter power output ($P_t$), transmitting antenna beam directivity ($G_t$) and free space (i.e. slant range) loss factor.
- $G_r^{\text{up}}$, $G_r^{\text{down}}$ are the up and down-looking receiving antenna patterns, written in terms of polar angles $\{\theta, \phi\}$ fixed to the respective antenna body frames.
- $\mathcal{X}_{\text{scat}}(\tau_r, \omega)$ is the scattered GPS waveform proper, defined as:

$$\mathcal{X}_{\text{scat}}(\tau_r, \omega) = \int_{-\infty}^{\infty} \frac{\sigma^0(\vec{r}) \mathcal{X}(\tau_r, \omega, \vec{r})}{4\pi R^2(\vec{r})} d^2 r \quad (3.3.3)$$

It expresses the sum of all re-radiated power contributions from the surface, weighted by the GPS autocorrelation function, $\mathcal{X}(\tau_r, \omega, \vec{r})$.

The scattered GPS waveform is obtained from the quotient between the scattered correlation power in (3.3.2) and the peak direct power in (3.3.1) as:

$$\mathcal{X}_{\text{scat}}(\tau_r, \omega) = \frac{\mathcal{X}_{\text{scat}}(\tau_r, \omega)}{\mathcal{X}_{\text{direct}}(\tau_r, \omega)} \frac{P_{\text{direct}}(\tau_r, \omega)}{P_{\text{reflected}}(\tau_r, \omega)} \quad (3.3.4)$$
Where $\alpha_{AGC} = \frac{\text{AGC}_{\text{up}}}{\text{AGC}_{\text{down}}}$ is the ratio of AGC factors. To form the bistatic waveform, we compute the ratio between power measurements collected by the up and down-looking antennas and correct it for the receiving antenna pattern (see Appendix B).

4 The target: The sea ice cover

The term cryosphere applies to that portion of the Earth where water can be found in its solid state, either as snow, continental ice, sea ice or permafrost. Among the cryospheric forms, the sea ice cover receives special attention (Figure I.11). With a thickness smaller than that of the continental ice sheets, it has a profound influence on the climate system: it modulates the amount of incoming solar and outgoing terrestrial radiation, it is an effective heat insulator, and it is a major agent in the circulation of fresh water in the oceans [Washington & Parkinson, 1986].

![Figure I.11 – Sea ice cover in the Northern Hemisphere: Ice analyses from the US Naval/National Ice Center [Dedrick et al., 2001], showing the summer-winter variability in Arctic sea ice extent.](image)

4.1 Structural and dielectric properties

The physical properties of the target determine the techniques that will be effective for its detection [Haykin et al., 1994]. Sea ice can be roughly divided into five major thickness/age categories: new, young, thin first-year, first-year and multiyear ice (see
Figure I.12, [WMO,1970]). Since its formation from crystal suspensions as young ice, the process of growth into first-year and multi-year ice is one of steady desalination, thickness increase, and surface erosion. The thickness of the overlying snow layer on top of the sea ice increases in general as the sea ice grows. In this section, we will review the development stages of sea ice and the basic terminology required for its classification.

**Formation**

Sea ice formation into new ice [see Figure I.13(a)] begins at the sea surface with a suspension of ice crystals known as *frazil* [Tucker et al., 1992]. When ice forms in calm seas, the frazil rises to form an unconsolidated layer of crystals known as *grease ice*, which stabilizes the sea surface and suppresses the formation of capillary waves in the presence of wind. Continued freezing results in a smooth, thin, elastic ice known as *dark nilas*. Consolidation progresses by water crystallization, with a resulting increase in salinity of the remaining liquid. Some of the saline brine is forced out of the ice mass to the sea beneath and to the surface. The remainder of brine is trapped within the ice in
vertically elongated pockets. Continued ice growth occurs on the bottom of the ice sheet. Wave action causes the ice to lump and form small rounded floes called *pancakes*. During calm periods, the pancakes will bond together forming an ice sheet, with continued growth on the bottom of the sheet.

*Growth*

The process of growth into first-year ice [see Figure I.13(b)] is a process of continuing brine drainage, thickness and free-board increase. The increasingly air-filled brine pockets change the optical appearance of ice from dark to *gray ice* to *white ice*. Along with the evolution of the optical appearance are changes in the electromagnetic and mechanical properties of the ice slab.

*Deformation*

The ice surface, far from retaining the relatively smooth structure originally formed, will undergo continual change under forcing from wind, waves and ocean currents. The most notable sea ice deformation features are ridges, rubble fields, ice rafts and leads [see Figure I.13(d)]. When bulk salinity is high, the ice is elastic to first buckle under compression and then break forming *ice rafts*, which can produce meter-scale roughness in marginal ice zones [Tucker et al., 1992]. When bulk salinity is low, the ice will fracture under compression forming piles of rubble blocks along failure lines, with wind induced snow dunes and thick snowdrifts.

*Melt*

At the end of the growth season, increasing temperatures will drive the melting of the sea ice cover. The absorption of solar radiation is governed by the surface albedo, which is
mainly controlled by the snow cover. Bare sea disintegrates more quickly than snow-covered sea ice, and this pattern of differential melting will result in the appearance of melt ponds, hummocks, drainage channels and weathered ridges [see Figure I.13(e)] [Tucker et al., 1992]. Some first-year ice survives the summer melt, becoming thick multyear ice, with a top layer transformed into a porous, low salinity cover, and a surface relief that becomes increasingly modulated by snow deposition and wind erosion [see Figure I.13(c)].

![Figure I.13 – Development stages of sea ice]( Photos courtesy of the Canadian Ice Service and the JPL Polar Oceanography Group)

**Dielectric properties of sea ice**

The electrical properties of sea ice are relevant, along with structural features such as surface roughness and the distribution of inhomogeneities, in understanding how it interacts with the incident electromagnetic radiation [Onstott, 1979]. At frequencies higher than about 1 GHz, the propagation of electromagnetic energy into sea ice is best
characterized by an effective complex permittivity $\epsilon = \epsilon' + i \epsilon''$, where $\epsilon'$ is the dielectric constant and $\epsilon''$ the dielectric loss of the medium. The former is related to the speed of propagation, while the latter controls the rate of attenuation of electromagnetic energy flow.

Any boundary between media with different dielectric properties will affect the propagation of electromagnetic radiation incident on it. In the analysis of scattering from sea ice, one has to consider a transmitted wave incident on a medium that consists of several layers with different complex permittivities: typical 1GHz values are $\epsilon = 1$ for air, $\epsilon = 1.5$ for snow, $\epsilon = 3.5 + i 0.1$ for sea ice and $\epsilon = 80 + i 30$ for sea water. When a wave hits the uppermost boundary (see Figure I.14), a portion of the incident energy is scattered upwards and the rest is transmitted into the lower layer. If the lower layer is homogeneous, the problem then reduces to surface scattering. But if the lower layer is inhomogeneous, then a portion of the transmitted wave may be scattered backwards and give way to volume scattering [Ulaby et al., 1986]. The more abrupt the change in permittivity, the stronger the scatter.

Figure I.14 – Interaction mechanisms for electromagnetic radiation
The dielectric constant of *pure ice* is $\epsilon' = 3.17$ and its dielectric loss $\epsilon'' = 0.001-0.01$ throughout the microwave range (1-1000 GHz) [Matzler & Wegmuller, 1987]. Because *sea ice* is a mixture of pure ice, brine pockets and air bubbles [Hallikainen & Winebrenner, 1992], its electrical properties have an added dependence on: i) *saline content*, which increases both the real and imaginary parts of the permittivity, and ii) *dielectric inhomogeneities*, which act as Rayleigh-Mie scattering centers that redistribute the incoming radiation, more effectively as the radiation wavelength in the medium approaches the typical size of the inhomogeneity.

Figure I.15 – Sea ice dielectric permittivity and penetration depth ([Ulaby et al., 1986])

The dielectric complex permittivity reported for different sea ice forms and radiation wavelengths is shown on the left panel in Figure I.15 [Ulaby et al., 1986]. In general, the permittivity contrasts among sea ice forms increase with the radiation wavelength. Measured values of the dielectric constant $\epsilon'$ for sea ice in the 1-40 GHz range fall between 2.5 and 8. The dielectric loss $\epsilon''$ ranges from less than 0.01 to more than 1.0, increasing with temperature, frequency and salinity [Vant el al., 1978]. The differences in permittivity are to be mainly attributed to the saline content and distribution of brine in the ice: the most saline ice forms (younger ice types) have higher permittivities (and
higher losses by absorption) than the less saline and older ice forms. The presence of vertically aligned brine pockets in thin ice types is responsible for a degree of dielectric anisotropy (i.e. larger V-pol permittivities, [Tucker et al., 1992]).

The effective depth of interaction, defined as the vertical extent of the column of sea ice that gives rise to effective power contributions, is determined by the amount of incident energy available for scattering (in active sensors) or for thermal emission (in passive sensors). There are two mechanisms responsible for the attenuation of radiation in sea ice: one is scattering (surface and volume) and the other is absorption (via dielectric losses and into Joule’s heat). The effective depth of penetration $\delta_p$, defined as that length before the incident power has decayed by a factor of $1/e$ due to absorption, can be expressed as [Hallikainen & Winebrenner, 1994]:

$$\delta_p = \frac{\lambda}{4\pi\sqrt{\varepsilon}} \frac{\lambda\sqrt{\varepsilon'}}{2\pi\varepsilon''}$$

The sea ice penetration depth is shown on the right panel in Figure I.15. Across a wide range of microwave frequencies, the penetration depth into saline (younger) ice types will be of the order of a wavelength, excluding the possibility of sub-surface scattering effects. For less saline (older, i.e. brine cleared) multiyear ice, the penetration depth will be $\sim 10\lambda$, giving way to potentially substantial volume scattering and scattering/emission from underlying boundaries. Typical values of the penetration depth caused by absorption in dry snow at frequencies in the 1-40 GHz range are about $\sim 100\lambda$ [Hallikainen, 1992]. For the typical pocket size and fraction volume encountered in sea ice, Rayleigh scattering considerations guarantee that volume scattering will be negligible below 24 GHz for first year ice and 1.5 GHz for multiyear ice [Vant et al., 1978]. Typical values for the penetration depth due to volume scattering in a layer of dry snow are $\delta_{scat} \sim 0.1-1$ m [Wiesmann et al., 1998], which indicate that this effect is strong enough at high frequencies for passive microwave signatures to respond directly to the snow cover.
4.2 Current observation techniques

Satellite sensors provide a convenient way to monitor the vast expanse of sea ice in the polar regions. As a response to the need for global estimates of parameters like the extent and thickness of sea ice, a suite of spaceborne sensors have been deployed that routinely provide synoptic observations of microwave-to-optical backscatter and emission signatures of the polar caps (see Table I.1).

Table I.1 – Operational polar dedicated spaceborne sensors in 2007

<table>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AMSR</td>
<td>Aqua, ADEOS</td>
<td>Polarimetric spectral radiances at frequency bands between 6 and 89 GHz</td>
<td>sea ice concentration, classification, snow cover</td>
</tr>
<tr>
<td>Advanced Microwave Scanning Radiometer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSM/I</td>
<td>DMSP</td>
<td>Polarimetric spectral radiances at frequency bands between 35 and 85 GHz</td>
<td>sea ice concentration, classification, snow cover</td>
</tr>
<tr>
<td>Special Sensor Microwave Imager</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AVHRR</td>
<td>POES</td>
<td>Spectral radiances at 6 frequency bands between 0.6 μm and 12 μm</td>
<td>sea ice extent, snow cover</td>
</tr>
<tr>
<td>Advanced Very High Resolution Radiometer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MODIS</td>
<td>Aqua, Terra</td>
<td>Spectral radiances at 36 frequency bands between 0.4 μm and 14 μm</td>
<td>sea ice extent, snow cover</td>
</tr>
<tr>
<td>Moderate Resolution Imaging Spectrometer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GLAS</td>
<td>Icesat</td>
<td>Laser waveforms</td>
<td>ice altimetry, surface roughness</td>
</tr>
<tr>
<td>Geoscience Laser Altimeter System</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At present, the characterization of sea ice types is done primarily with microwave sensors, which contrary to visible or near-infrared instruments, can operate at night and in all-weather conditions. With an imaging radar, the microwave energy backscattered from the scene is related in pattern and intensity to a certain ice class. By combining
backscatter with measurements of brightness temperatures, the ability to classify sea ice improves further [Beaven, 1995]. The currently operational polar observing satellites allow the determination of the extent of the sea ice cover, a classification of sea ice, and delimiting the snow cover over large areas, albeit with a fairly large error for some of the parameters, depending on factors such as time of the year and atmospheric conditions.

4.2.1 Microwave backscatter

Radar sensors, such as scatterometers or the higher resolution synthetic aperture radar (SAR), use electromagnetic pulses at low microwave frequencies (L, C and X bands) to excite the Earth’s surface. The backscatter cross-section is calculated as:

$$\sigma^0 = \frac{P_r^2}{P_t^2 / (4\pi R^2)}$$

Where $P_r$ and $P_t$ are the received and transmitted power and $R$ is the distance to the surface.

Figure I.16 – RADARSAT (5.3 GHz HH) SAR image of Barrow, AK in March of 2003 (Backscatter angles range from 20 to 50 degrees)

The main geophysical parameters that shape the backscatter cross-section, $\sigma^0$, are a) the surface dielectric permittivity, which acts on the absolute backscatter level, b) the surface roughness, which influences the angular distribution of backscatter, and c) the presence of
inhomogeneities, whose contribution to volume scattering both raises the level and smooths the angular distribution of backscatter [Onstott, 1992]. In general, the backscatter cross-section increases with surface roughness and permittivity, but relating backscatter cross-sections to surface properties is still subject to ambiguities (See Figure 1.16):

i)  **Roughness vs. permittivity**: a rough but weakly scattering surface (i.e. sea ice) will have the same cross-section as a smoother but strongly scattering surface (i.e. sea water under wave-wind action).

ii) **Melt season**: during the summer season, a thin layer of melt water will effectively mask the signatures of any underlying sea ice types.

iii) **Snow cover**: considered transparent to microwaves when dry, the snow cover remains a factor of backscatter variability and classification error.

Ambiguities notwithstanding, each sea ice type is found to have a characteristic backscatter signature:

- **Multiyear ice** backscatter is a function of both surface and volume scattering, the latter due to increased penetration depths with lower salinity and a denser population of inhomogeneities.

- **First year ice** is very saline. If features a smooth surface if grown under calm conditions, or a rough surface when subject to deformation processes. The backscatter from first year ice (both smooth and deformed) is dominated by surface contributions. The snow cover is thinner than in multiyear ice (~0.1 m).

- The key characteristic of the backscatter from **new ice** is its weakness. The formation of new ice impedes the formation of waves, producing a surface that is smooth to the radar and enhances forward scatter.
To characterize sea ice, SAR imagery is set against an intensity-based scale with empirically fixed thresholds that can statistically separate between multiyear ice, rough first year ice, smooth first year ice and open water during the polar winter months [Kwok & Tsatsoulis, 1998] (see Table I.2).

Table I.2 – C-Band SAR mean backscatter intensities (5.3 GHz VV) [Kwok et al., 1992]

<table>
<thead>
<tr>
<th>Ice Type</th>
<th>Mean $\sigma^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiyear</td>
<td>-10 dB</td>
</tr>
<tr>
<td>Deformed FYI</td>
<td>-14 dB</td>
</tr>
<tr>
<td>Smooth FYI</td>
<td>-17 dB</td>
</tr>
<tr>
<td>New ice/open water</td>
<td>-22 dB</td>
</tr>
</tbody>
</table>

The backscatter cross-section also depends on the polarization of the incident wave, allowing the use of this information to better define the nature and structure of the surface [Drinkwater et al., 1991]. Measurements show that the polarization ratio V/H of open water is greater than that of sea ice. The frequency of the incident wave plays a major role in the interaction with the surface, in that it determines the depth of penetration and the dominant mechanism of surface scattering. The complexity and wide variety of natural surfaces make it extremely hard to exactly model the backscatter behavior as a function of polarization, frequency and incidence angle.

4.2.2 Microwave emission

Microwave radiometers (such as AMSR or SSM/I in Table I.1) directly observe the thermal emissions from the surface at frequencies from C-band to millimeter wavelength. This spectral range has to compensate for greater atmospheric absorption, generally by including a channel centered at the 21 GHz absorption line of water vapor for corrections. The measured radiometric power or radiance is a function of the surface physical
temperature $T$ and its directional emissivity $\varepsilon(\Theta)$, the latter in turn related to the bistatic radar cross-section (see Section 4.4 in Chapter II). The radiance is usually expressed as a brightness temperature $T_B$, modeled as (see Appendix A):

$$T_B = \frac{P_r}{k_B \Delta \nu} = \frac{1}{4\pi} \int_{\Omega_{\text{recv}}} [T \in (\Theta)] G(\Theta, \Phi) d\Omega$$

Where $P_r$ is the radiometric power, $k_B$ the Boltzmann constant, $\Delta \nu$ the frequency bandwidth of the radiometric channel, and $G(\Theta, \Phi)$ the receiver antenna gain pattern.

19 GHz V 19 GHz H 37 GHz V

Figure I.17 – SSM/I images of Barrow, AK in March of 2003

As with the radar backscatter, the major features in microwave emissivities from sea ice generally involve strong versus weak (surface and volume) scattering and the large difference in dielectric permittivity between sea ice and open water (see Figure I.17, the sea ice cover north of Point Barrow is invariably warmer than the water opening extending along the western coast). The algorithms that characterize sea ice using microwave radiances are based on surface emissivity differences at separate frequencies and polarizations (i.e. spectral gradient and polarization ratios) and they rely on a small set of empirically determined radiances that correspond to species representative of “pure” multiyear ice, first year ice and open water ([Cavalieri et al., 1984], [Thomas, 1993]).
As shown on the left panel in Figure I.18, open water (and new ice) is characterized by a strong polarization, whereas the difference between vertically and horizontally polarized radiances for other sea ice types remains small. On the basis of this difference, currently operational algorithms parameterize the extent of polar sea ice in terms of the polarization ratio $PR$ at 19 GHz defined by:

$$PR = \frac{T_h(19V) - T_h(19H)}{T_h(19V) + T_h(19H)}$$

Observe too that, while the emissivity of first year ice remains quite flat throughout the microwave spectrum, that of multiyear ice increases steadily, mainly due to stronger volume scattering contributions from inhomogeneities and snow.

![Figure I.18](image)

Figure I.18 – Measured microwave emissivities of sea ice [Eppler et al., 1992] and distribution of sea ice species in PR-GR space [Comiso et al., 1997]
(MY is multiyear ice, FY is first year ice, NI is new ice and OW is water)

The principal basis thus for distinguishing among first and multiyear ice lies in their differing spectral emissivity gradients, a feature best parameterized by the spectral gradient ratio $GR$ defined by:

$$GR = \frac{T_h(37V) - T_h(19V)}{T_h(37V) + T_h(19V)}$$
Satellite radiometer data can at best determine the fractional coverage of three types of polar surfaces: open water, FY ice and MY ice, provided that the typical polarization (PR) and spectral gradient (GR) ratios of “pure” winter ice species have been accurately determined in advance (see right panel in Figure I.18) [Cavalieri et al., 1984]. The current knowledge of microwave sea ice signatures allows the determination of the first year, multiyear and open water concentrations, still subject to errors related to seasonal and geographical variations in “pure” species radiances and the presence of thin ice types.

4.2.3 Optical signatures

Visual reconnaissance from aircraft was initially used to map sea ice distributions but this method suffered from limited range and subjective interpretation techniques [Kwok & Tsatsoulis, 1998]. As early as 1972, the launch of the NOAA polar orbiting satellites led to ice analyses derived from visible and infrared imagery. Imaging spectrometers (such as AVHRR or MODIS in Table I.1) measure reflectances (i.e. solar albedos) or radiances, depending on whether observations are made during day or at night or in the visible or thermal infrared bands, and are always subject to their inability to penetrate the persistent polar cloud cover. The basis of ice classification in the optical spectrum relies, as in the two previous cases, in the natural segmentation of surface types into clusters in the space of measurements [Massom & Comiso, 1994], and provides the best delineation of sea ice extent (see Figure I.19), when unimpeded by clouds.

<table>
<thead>
<tr>
<th>Ice type</th>
<th>Albedo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open water</td>
<td>0.05-0.10</td>
</tr>
<tr>
<td>Sea ice</td>
<td>0.30-0.40</td>
</tr>
<tr>
<td>Old snow</td>
<td>0.50-0.70</td>
</tr>
<tr>
<td>Fresh snow</td>
<td>0.80-0.90</td>
</tr>
</tbody>
</table>
4.2.4 Multisensor approach

In an attempt to increase the level of information that can be exploited in segmenting the data, present trends point at multisensor approaches to sea ice classification using simultaneous microwave and optical satellite observations, ([Gray et al., 1992], [Steffen & Schweiger, 1990], [Drinkwater et al., 1991], [Remund, 2000]).

Table I.4 – Summary of sea ice emission/backscatter signatures

(\(GR = \) gradient ratio, \(PR = \) polarization ration, \(T_B = \) brightness temperature)

<table>
<thead>
<tr>
<th>Surface Type</th>
<th>Microwave</th>
<th>Optical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Backscatter</td>
<td>Emission</td>
</tr>
<tr>
<td>Open water</td>
<td>Wind dependent</td>
<td>Positive GR</td>
</tr>
<tr>
<td>New ice</td>
<td>Low</td>
<td>Zero GR</td>
</tr>
<tr>
<td>First Year Ice</td>
<td>Medium</td>
<td>Zero PR</td>
</tr>
<tr>
<td>Multiyear Ice</td>
<td>High</td>
<td>Negative GR</td>
</tr>
</tbody>
</table>

Figure I.19 – AVHRR visible and infrared images of Barrow, AK in March of 2003
Table I.4 shows a summary of the sea ice emission and backscatter signatures, which taken all together, provide a more reliable classification of sea ice forms than is possible with individual sensors.

5 Summary

While there is a substantial body of work on forward modeling of electromagnetic scattering from sea ice, the inversion of sea ice parameters from satellite data is based on a collection of rather ad hoc techniques, which in most cases require a priori statistical knowledge of the ice conditions [Winebrenner, 1992]. Still, the utilization of a base of knowledge about visible, passive and active microwave signatures of sea ice for its classification is encumbered by internal ambiguities and seasonal variability factors. More theoretical approaches to the interpretation of scattering from sea ice have made attempts to test observed microwave and optical signatures against model predictions and investigate their dependence on frequency and incidence angle ([Eom, 1981], [Kim, 1984], [Drinkwater & Crocker, 1988], [Winebrenner et al., 1989], [Fetterer et al., 1992], [Grenfell, 1993]), although the understanding of the physics of backscatter and emission from sea ice remains incomplete.

The analysis of bistatic scattering of GPS signals affords an excellent opportunity to further on the theoretical study of microwave signatures of sea ice. The pulse limited structure of the GPS signal allows establishing a linear relation between the angular pattern of the sea ice electromagnetic scattering cross-section (itself a function of the physical properties of the target) and the shape of the scattered power waveform. In general, both surface and volume contributions are present in scattering or emission from sea ice, but the long wavelength of the GPS radiation guarantees that surface scattering effects remain the dominant mechanism of interaction, allowing to neglect volume effects.
that would otherwise arise from an overlying snow layer or the sea ice own internal inhomogeneities.
CHAPTER II – Theory
1 Introduction

Remote sensing applications call for the development of models that are able to predict the scattering cross-section of a statistically rough surface under electromagnetic excitation. The problem of scattering from rough boundaries cannot be solved exactly but it admits approximate analytic solutions, whose construction and limitations constitute the main purpose of this chapter. The two methods most widely used are the classical *Small Perturbation Method* (SPM) and the *Kirchhoff Approximation* (KA). Initially, we will dwell on the building principles of these classical approaches, to move on to the *Small Slope Approximation* (SSA), which in principle should afford a better understanding of the effects of all scales of roughness on the scattering signature in a complete model.

2 Statistically rough surfaces

A random surface is characterized by its vertical height distribution and a transverse correlation function. The surface height function $\xi(x,y)$ is regarded as a random field with a pointwise height distribution given by the probability density function $PDF_h[\xi(x,y)]$, which gives the probability of finding any point on the surface at a distance between $h$ and $h+dh$ away from the mean reference surface. Most literature assumes that random surface height distributions are Gaussian, i.e.

$$PDF_h(\xi) = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2}(\xi)^2}$$

where $\sigma$ is the root mean square (RMS) height of the surface. Evidence for the validity of this assumption is sometimes conflicting, depending on the nature of the surface, and alternative models are devised based on non-gaussian height distributions [Eom & Fung, 1983]. The complete statistical description of the surface random field can be made
arbitrarily complex, but it is often simplified to the description of its second order moments, for which the joint statistics at two different locations are solely needed [Rytov et al., 1989]. An important process that is completely described in this manner is the (stationary and ergodic) gaussian random process, with a joint (bivariate) gaussian probability distribution given by,

\[
\text{PDF}_b(\xi, \xi') = \frac{1}{2\pi\sigma^2 \sqrt{1 - \rho^2}} e^{-\frac{\xi^2 + \xi'^2 - 2\rho\xi\xi'}{2\sigma^2(1-\rho^2)}} \quad (2.1)
\]

Where \( \rho(\vec{r}, \vec{r}') \) is the normalized autocorrelation function of surface heights:

\[
\rho_0(\vec{r}, \vec{r}') = \langle \xi(\vec{r})\xi(\vec{r} + \vec{r}') \rangle / \sigma^2 \quad (2.2)
\]

\[
\sigma^2 = \langle \xi(\vec{r})\xi(\vec{r}) \rangle
\]

An alternative statistical description of this type of surface is given by the power spectral density (PSD or “spectrum”) of roughness \( S(k) \), defined as the Fourier transform of the autocorrelation function:

\[
S(\vec{k}) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} d^2r \rho(\vec{r}) e^{i\vec{k} \cdot \vec{r}}
\]

With

\[
\rho(\vec{r}) = \langle \xi(\vec{r} + \vec{r}')\xi(\vec{r} + \vec{r}') \rangle = \iint_{-\infty}^{\infty} d^2k S(\vec{k}) e^{i\vec{k} \cdot \vec{r}} \quad (2.3)
\]

The Fourier transform of the random surface height function is, by the convention chosen in this work:

\[
\tilde{\xi}(\vec{k}) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} d^2r \xi(\vec{r}) e^{-i\vec{k} \cdot \vec{r}}
\]

With

\[
\xi(\vec{r}) = \iint_{-\infty}^{\infty} d^2k \tilde{\xi}(\vec{k}) e^{i\vec{k} \cdot \vec{r}} \quad (2.4)
\]

And the ensemble average of the roughness Fourier amplitudes:

\[
\langle \tilde{\xi}(\vec{k})\tilde{\xi}^*(\vec{k}') \rangle = S(\vec{k}) \delta(\vec{k} - \vec{k}') \quad (2.5)
\]

With

\[
\langle \tilde{\xi}(\vec{k})\tilde{\xi}^*(\vec{k}) \rangle = S(\vec{k}) A/(2\pi)^2
\]
where $A$ is the surface area. Another useful relation is given by the Fourier transform of the PDF of heights [Vanmarcke, 1984], called the characteristic function $X(q)$, defined as:

$$
X_h(q) = \int PDF_h(\xi)e^{iq\xi}d\xi = \langle e^{iq\xi} \rangle \quad PDF_h(\xi) = \frac{1}{2\pi} \int X_h(q)e^{-iq\xi}dq \quad (2.6)
$$

Several assumptions are often implicit in statistical theories of randomly rough surfaces, which we will also adopt here onwards: a) Gaussian distributed heights, b) isotropy (i.e. rotational invariance of statistics), c) stationarity (i.e. translational invariance of statistics), and d) ergodicity (i.e. ensemble averages that remain equivalent to spatial or temporal averages).

Since gradients are linear operators, the probability distribution of surface slopes ($\nabla h = \tan \beta$) of a random gaussian surface remains gaussian, provided that the surface is differentiable, and its variance (i.e. the mean square slope, mss) is determined by the second derivative of the autocorrelation function evaluated at the origin (see Appen. K):

$$
PDF_{\xi}(\tan \beta) = \frac{1}{2\pi \text{mss}} e^{\frac{-\tan^2 \beta}{2\text{mss}}} \quad \text{mss} = \langle \tan^2 \beta \rangle = -\frac{\partial^2 \rho(r)}{\partial r^2} \bigg|_{r=0} \quad (2.7)
$$

A few common statistical two-dimensional and isotropic roughness models are listed next, along with their parameters RMS height ($\sigma$) and correlation length (L). We will make extensive use of them in subsequent discussions.

A) Gaussian correlated surface

The autocorrelation and spectrum of a Gaussian correlated surface (Fig. II.1) are given as:

$$
\rho(r; \sigma, L) = \langle \xi(\vec{r} + \vec{r'})\xi(\vec{r'}) \rangle = \sigma^2 e^{-(r/L)^2} \quad S(k; \sigma, L) = \frac{(\sigma L)^2}{4\pi} e^{-((kL)^2/2)} \quad (2.8)
$$

The two-dimensional mean square slope of this surface:

$$
\sigma^2_{\nabla h} = \left\langle \nabla h^2 + \nabla h^2 \right\rangle = 2\pi \int_0^\infty k^2 \sigma^2 PDF_{\nabla h}(\gamma; \sigma, L) = 2\pi \int_0^\infty k^3 dk S(k; \sigma, L) = 2\text{mss}
$$
With
\[ m_{ss} = \left. \frac{\partial^2 \rho(r)}{\partial r^2} \right|_{r=0} = 2(\sigma/L)^2 \]  
(2.9)

\[ \rho(r; \sigma, L) = \langle \xi(\vec{r} + \vec{r}')\xi(\vec{r}') \rangle = \sigma^2 e^{-r/L}, \quad S(k; \sigma, L) = \frac{(\sigma L)^2}{2\pi} \left(1 + (kL)^2\right)^{-3/2} \]  
(2.10)

Figure II.1 – Gaussian correlated surface: spectrum (left), autocorrelation (right) for \( \sigma=0.08\) m, \( L=5.5\) m with KAGO quadratic approximant

**B) Exponentially correlated surface**

The autocorrelation and spectrum of an exponentially correlated surface (Fig. II.2) are given by:

\[ \rho(r; \sigma, L) = \langle \xi(\vec{r} + \vec{r}')\xi(\vec{r}') \rangle = \sigma^2 e^{-r/L}, \quad S(k; \sigma, L) = \frac{(\sigma L)^2}{2\pi} \left(1 + (kL)^2\right)^{-3/2} \]  

Figure II.2 – Exponentially correlated surface: spectrum (left), autocorrelation (right) for \( \sigma=0.08\) m, \( L=5.5\) m with KAGO linear approximant

Unfortunately, exponentially correlated surfaces are not differentiable, and thus constitute very poor models of roughness. For a surface of this type to become differentiable (i.e. have an autocorrelation function with zero first derivative and finite second derivative at the origin), it will suffice to filter out the highest frequency components of roughness, as the next surface model does.
C) Expo-gaussian correlated surface

The autocorrelation function of an expo-gaussian correlated surface (Fig. II.3) is given as:

\[ \rho(r; \sigma, L_1, L_2) = \sigma^2 e^{-r^2/(L_1^2)} \approx \begin{cases} \sigma^2 e^{-r^2/(L_1^2)} & r << L_2 \\ e^{r^2/L_2} & r >> L_2 \end{cases} \]  

(2.11)

This surface model is exponentially correlated at large scales and gaussian correlated at small scales. The parameter \( L_1 \) is the large scale exponential correlation length, and the parameter \( L_2 \) marks the length scale where the transition to a gaussian surface occurs. The two-dimensional mean square slope of this surface is very sensitive to the choice of the scale transition parameter \( L_2 \):

\[ \sigma_{vh}^2 = 2\pi \int_0^\infty y^3 dPDF_{vh} (\gamma; \sigma, L_1, L_2) = 2\pi \int_0^\infty k^3 dk S(k; \sigma, L_1, L_2) = 2mss \]

\[ mss = -\frac{\partial^2 \rho(r)}{\partial r^2} \bigg|_{r=0} = 2\sigma^2 / L_1 L_2 \]  

(2.12)

Unlike the previous examples, the spectrum of an expo-gaussian correlated surface does not admit a closed form expression and must be computed numerically.

On the grounds of large surface deviations, as will be shown later in Section 5.1, the scattering cross-section in the Kirchhoff approach (geometric optics, KAGO) introduces a quadratic approximant to the surface autocorrelation in the midst of its derivation, establishing a direct connection between the KAGO cross-section and the surface mean square slope. From a Taylor expansion of the autocorrelation:

\[ \rho(r) = 1 + \frac{\partial \rho(r)}{\partial r} \bigg|_{r=0} r + \frac{1}{2} \frac{\partial^2 \rho(r)}{\partial r^2} \bigg|_{r=0} r^2 + ... \approx 1 - \frac{mss}{2} r^2 \]  

(2.13)

Since the linear term must vanish for differentiable surfaces. This quadratic approximation, which seems perfectly justifiable for gaussian correlated surfaces (see Figure II.1), cannot be used for exponentially correlated surfaces, due to a non-vanishing
linear term in the expansion (see Figure II.2). For expo-gaussian correlated surfaces, the scale transition parameter $L_2$ determines how well the quadratic approximant will approach the autocorrelation function. In general, surfaces with larger amounts of small scale roughness (i.e. smaller $L_2$) have worse quadratic representations (see Figure II.3), since small scale slopes are being attributed to KAGO “large scale” features, forcing KAGO to work outside of its domain of validity (see Section 6.1).

![Figure II.3](image)

3 Scattering cross-section

Any interface separating two media with different electric or magnetic properties acts as an obstacle to the propagation of the radiation. If the interface is perfectly flat and infinitely large, the field that emerges in response to an incident plane wave will consist of reflected and transmitted plane waves along well defined directions and with amplitude relations given by the Fresnel coefficients. But if the surface is rough, incident energy...
will propagate in directions other than those predicted by Snell’s laws. The normalized scattering cross-section, $\sigma^0$, which describes the angular distribution of scattered radiation, is defined as the ratio of the scattered power flux density along a direction ($\theta$, $\phi$) divided by the isotropic redistribution of power incident on the surface. In terms of the illuminated area $A$, the incident and scattered fields $E_i, E_s$ along polarization unit vectors $p_0, p$, and the distance $R$ from the surface to the observation point, the scattering cross-section is given by:

$$\sigma^0_{p_0} (\theta,0;\theta,\phi) = \frac{4\pi R^2}{A} \frac{|\hat{p} \cdot E_i|^2}{|\hat{p}_0 \cdot E_i|^2}$$  \hspace{1cm} (3.1)$$

Figure II.4 – Scattering on a rough surface
(with input and output polarization vectors $p_0, p$)

The main task in the theory of wave scattering from rough surfaces is the calculation of the scattered fields. The following section provides some formal relations to help translate the scattered field solutions into normalized scattering cross-sections, introducing concepts such as scattering amplitudes (for spherical wave solutions) and Fourier amplitudes (for plane wave solutions).

3.1 The scattered fields

Assume an incident plane wave field of unit amplitude, $|E_i(\theta,0)| = 1$. If the field $E_s$ scattered from the surface into a direction specified by polar angles $\theta$ and $\phi$ is written in the form of an outgoing (radial) spherical wave:
\[ E_s(R, \theta, \phi) = F(\theta, \phi) \frac{e^{i\kappa R}}{R} \]  

(3.1.1)

Where \( F(\theta, \phi) \) is the \textit{scattering amplitude}, then from (3.1):

\[ \sigma^0(\mathbf{n}_{inc}, \mathbf{n}_{scat}) = \frac{4\pi}{A} \left| F(\mathbf{n}_{inc}, \mathbf{n}_{scat}) \right|^2 \]  

(3.1.2)

But if the scattered field is written in the form of an expansion in plane wave harmonics (i.e. from “analytically continued” fields after boundary conditions: Rayleigh hypothesis, see Appendix H3):

\[ E_s(R, \theta, \phi) = \iint \tilde{E}_s(\mathbf{k}_\perp) e^{i(\mathbf{k}_\perp \cdot \mathbf{z})} d^2 \mathbf{k}_\perp \]  

(3.1.3)

Where \( \tilde{E}_s(\mathbf{k}_\perp) \) are the \textit{Fourier amplitudes} of the scattered field, then by matching (3.1.1) and (3.1.3) at \( z = 0 \) (see Appendix E):

\[ F(\theta, \phi) = -i 2\pi \kappa_z \tilde{E}_s(\mathbf{k}_\perp) \]  

(3.1.4)

Whereby:

\[ \sigma^0(\mathbf{n}_{inc}, \mathbf{n}_{scat}) = \frac{4\pi}{A} \left( 2\pi \kappa_z \right)^2 \left\langle \left| \tilde{E}_s(\mathbf{k}_\perp) \right|^2 \right\rangle \]  

(3.1.5)

The total field \( E_s \) scattered from the surface can be considered as the sum of two components: a coherent (mean or specular) and an incoherent part (diffuse or fluctuating) [Beckmann & Spizzichino, 1963],

\[ E_s = \langle E_s \rangle + \Delta E_s \quad \text{with} \quad \langle \Delta E_s \rangle = 0 \]  

(3.1.6)

The total scattered intensity is the sum of the coherent and incoherent intensities:

\[ I = \langle E_s E_s^* \rangle = \left| \langle E_s \rangle \right|^2 + \left| \langle \Delta E_s \rangle \right|^2 \]

And the bistatic scattering cross-section is decomposed into coherent and incoherent parts, which a power detector in bistatic geometry measures simultaneously:

\[ \sigma^0 = \frac{4\pi R^2}{A} \left| \langle E_s \rangle \right|^2 = \frac{4\pi R^2}{A} \left| \langle E_{inc} \rangle \right|^2 + \frac{4\pi R^2}{A} \left| \langle \Delta E_{inc} \rangle \right|^2 = \sigma^0_{coh} + \sigma^0_{incoh} \]  

(3.1.7)
4 Diffraction theory

The actual calculation of the scattered fields rests on diffraction theory, which is associated with departures from geometrical (ray) optics caused by the finite wavelength of waves. The interaction between the electromagnetic fields and the surface is mediated by Maxwell equations [Jackson, 1998], which in the absence of external field sources (i.e. free space harmonic equations, with a factor $e^{i\omega t}$ everywhere implicit in the fields)

\[
\begin{align*}
\nabla \cdot \vec{B} &= 0 & \nabla \times \vec{H} + i\omega \epsilon \vec{E} &= 0 \\
\nabla \cdot \vec{D} &= 0 & \nabla \times \vec{E} - i\omega \mu \vec{H} &= 0
\end{align*}
\]

in MKS units (convention used in this document), where $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields, $\vec{D}$ and $\vec{B}$ the displacement and magnetic induction, and $\epsilon = \epsilon_r \epsilon_0$, $\mu = \mu_r \mu_0$, and $c^2 = 1/\epsilon_0 \mu_0$. Other authors prefer Gaussian units, so we include for reference the corresponding expressions:

\[
\begin{align*}
\nabla \cdot \vec{B} &= 0 & \nabla \times \vec{H} + i\omega \epsilon/c \vec{E} &= 0 \\
\nabla \cdot \vec{D} &= 0 & \nabla \times \vec{E} - i\omega \mu/c \vec{H} &= 0
\end{align*}
\]

Where $\epsilon = \epsilon_r$ and $\mu = \mu_r$. Under either convention, the Maxwell relations for electric and magnetic fields in a source-free, homogeneous medium lead to the time-independent Helmholtz wave equations:

\[
\begin{align*}
\nabla^2 \vec{E} + k^2 \vec{E} &= 0 \\
\nabla^2 \vec{H} + k^2 \vec{H} &= 0
\end{align*}
\]

Where $k = \omega \sqrt{\epsilon \mu} = \sqrt{\epsilon_r \omega/\epsilon c}$. Sometimes, the medium intrinsic impedance $Z = \sqrt{\mu/\epsilon}$ is used. The following relations apply to electromagnetic plane waves, and are used in the construction of specular field solutions (MKS units):

\[
\begin{align*}
\vec{E} &= \vec{E}_0 e^{i(k \cdot \vec{r} - \omega t)} & \vec{E} &= \frac{1}{\omega \epsilon} \vec{k} \times \vec{H} \\
\vec{H} &= \vec{H}_0 e^{i(k \cdot \vec{r} - \omega t)} & \vec{H} &= \frac{1}{\omega \mu} \vec{k} \times \vec{E}
\end{align*}
\]
The fact that electromagnetic fields must follow the Helmholtz (homogeneous, in the absence of external sources) wave equations leads to an analytical expression for the fields at any point within a source-free volume in terms of those same fields evaluated on the contouring surface: this is the field integral formula.

### 4.1 Scalar integral formula

To begin with, apply Stokes divergence theorem in a closed volume bounded by a (scattering) surface (see Figure II.5) to the vector functions $E \nabla \mathbf{G}$ and $G \nabla \mathbf{E}$, where $E$ and $G$ are the continuous scalar electric field and Green’s response functions, and subtract the results.

\[
\int \left( E(\vec{r}') \nabla'^2 G(\vec{r}', \vec{r}) - G(\vec{r}', \vec{r}) \nabla'^2 E(\vec{r}') \right) dV' = -\int \left( E(\vec{r}') \frac{\partial G(\vec{r}', \vec{r})}{\partial \vec{n}'} - G(\vec{r}', \vec{r}) \frac{\partial E(\vec{r}')}{\partial \vec{n}'} \right) dS'
\]

Where $\partial/\partial \vec{n}'$ is the normal derivative into the volume. Let $E$ and $G$ satisfy the homogeneous wave equation $(\nabla'^2 + k^2) \psi = 0$ within that volume (i.e. that no exciting sources exist around a bubble that encircles the observation point $\vec{r}$). Then the total field $E(\vec{r})$ within the volume is formulated as an integral of (as-of-yet unknown) surface contributions $E(\vec{r}')$ and $\partial E(\vec{r}')/\partial \vec{n}'$, known as the scalar Kirchhoff integral formula (see e.g. [Ishimaru, 1991]):

\[
E(\vec{r}) = \int_{\text{surface}} \left( E(\vec{r}') \frac{\partial G(\vec{r}', \vec{r})}{\partial \vec{n}'} - G(\vec{r}', \vec{r}) \frac{\partial E(\vec{r}')}{\partial \vec{n}'} \right) dS'
\]

(4.1.1)

Where the free space Green’s response function $G$ is given by:

\[
G(\vec{r}, \vec{r}') = \frac{e^{ik|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}
\]

(4.1.2)
4.2 Vector integral formula

There exists an equivalent formulation for the vector electromagnetic fields in a source-free volume. To derive the vector Kirchhoff integral formula, apply the scalar integral formula in (4.1.1) to each rectangular component of the electric field [Stratton, 1941]. Rearranging terms, making use of Maxwell equations and a few vector calculus theorems, yield the also-called Stratton-Chu integral equations:

$$
\vec{E}(\vec{r}) = \int_\Gamma i\omega \epsilon_0 \left( \vec{n} \times \vec{H} \right) G + \left( \vec{n} \times \vec{E} \right) \times \nabla G + \left( \vec{n} \cdot \vec{E} \right) \nabla G \, dS'
$$

$$
\vec{H}(\vec{r}) = -\int_\Gamma i\omega \mu_0 \left( \vec{n} \times \vec{E} \right) G - \left( \vec{n} \times \vec{H} \right) \times \nabla G - \left( \vec{n} \cdot \vec{H} \right) \nabla G \, dS'
$$

The fields in a domain enclosed by a connected surface are determined by these equations, as an integral function of unknown surface fields. A numerical solution to these equations is possible for bodies of dimensions comparable to the wavelength of the
radiation. In scattering from extended rough surfaces though, recourse is made to reasonable approximations for the surface fields. For instance, specializing to a far field scattering situation \( (r \to \infty, R = |r|, r = r' \) as in [Jackson, 1998], p. 485):

\[
G(\vec{r}, \vec{r}') \to \frac{e^{i\beta R}}{4\pi R} e^{-ikR}
\]

\[
\vec{E}_{\text{scat}} \to (-ik) \frac{e^{i\beta R}}{4\pi R} \hat{u}_k \times \left[ e^{-ikR} \left( Z_o \hat{u}_k \times (\hat{n} \times \vec{H}_{\text{stat}}) - (\hat{n} \times \vec{E}_{\text{scat}}) \right) \right] dS
\]

(4.2.2)

Once the tangential scattered fields \( E_{\text{scat}}, H_{\text{scat}} \) on the surface are known, the scattered field can be everywhere else computed. The equations of Maxwell allow one to solve for the magnetic field in (4.2.3), leaving an integro-differential equation for the scattered electric field alone that can be approached recursively.

4.3 Energy conservation and reciprocity

For any two arbitrary solutions \( \psi_1, \psi_2 \) of the scalar homogeneous Helmholtz equation, \( \nabla^2 \psi + k^2 \psi = 0 \), the following relation is fulfilled

\[
0 = \int_S \left( \psi_2(\vec{r}') \frac{\partial \psi_1(\vec{r}')}{\partial n'} - \psi_1(\vec{r}') \frac{\partial \psi_2(\vec{r}')}{\partial n'} \right) dS'
\]

(4.3.1)

for any surface \( S \) that delimits a volume \( V \) that does not contain any field sources (see Figure II.5). For a lossless medium (real \( k \) wavenumber) and a boundary along a horizontal plane at \( z=z_0 \):

\[
0 = \int_{z=z_0} \left( \psi_2(\vec{r}') \frac{\partial \psi_1(\vec{r}')}{\partial z'} - \psi_1(\vec{r}') \frac{\partial \psi_2(\vec{r}')}{\partial z'} \right) d^2r'
\]

(4.3.2a)

\[
0 = \int_{z=z_0} \left( \psi_2^*(\vec{r}') \frac{\partial \psi_1^*(\vec{r}')}{\partial z'} - \psi_1^*(\vec{r}') \frac{\partial \psi_2^*(\vec{r}')}{\partial z'} \right) d^2r'
\]

(4.3.2b)

Two possible solutions \( \psi_1, \psi_2 \) to the wave equation may be written in terms of the scattered field Fourier amplitudes as:
\[ \Psi_1(\vec{r}) = e^{i\vec{k}_{z1} \cdot \vec{r} - i\vec{k}_{z2} \cdot \vec{z}} + \int_{-\infty}^{\infty} d^2 \kappa \, \vec{E}_{\text{scat}}(\vec{k}_{z1}, \vec{\kappa}) e^{i\vec{\kappa} \cdot \vec{r} + i\vec{\kappa} \cdot \vec{z}} \]

\[ \Psi_2(\vec{r}) = e^{i\vec{k}_{z1} \cdot \vec{r} - i\vec{k}_{z2} \cdot \vec{z}} + \int_{-\infty}^{\infty} d^2 \kappa \, \vec{E}_{\text{scat}}(\vec{k}_{z2}, \vec{\kappa}) e^{i\vec{\kappa} \cdot \vec{r} + i\vec{\kappa} \cdot \vec{z}} \]  

(4.3.3)

Inserting these solutions into (4.3.2a) and integrating over the \( z = z_0 \) plane yields:

\[ k_{z2} \vec{E}_{\text{scat}}(\vec{k}_{z1}, \vec{k}_{z2}) = k_{z1} \vec{E}_{\text{scat}}(-\vec{k}_{z2}, -\vec{k}_{z1}) \]  

(4.3.4a)

This is known as the reciprocity condition, also in terms of the scattering amplitude:

\[ F(\vec{k}_{z1}, \vec{k}_{z2}) = F(-\vec{k}_{z2}, -\vec{k}_{z1}) \]  

(4.3.4b)

Inserting the field solutions (4.3.3) into (4.3.2b) and integrating over the \( z = z_0 \) plane yields:

\[ \delta(\vec{k}_{z1} - \vec{k}_{z2}) = \frac{2\kappa_2}{k_{z1} + k_{z2}} \vec{E}_{\text{scat}}(\vec{k}_{z1}, \vec{\kappa}) \vec{E}_{\text{scat}}^*(\vec{k}_{z2}, \vec{\kappa}) d^2 \kappa \]  

(4.3.5)

This is known as the unitarity relation for the scattered field Fourier amplitudes. To arrive at the law of energy conservation, choose \( \vec{k}_{z1} = \vec{k}_{z2} \) and proceed to:

\[ \delta(\vec{0}) = \int \frac{\kappa_z}{k_{z1}} \left| \vec{E}_{\text{scat}}(\vec{k}_{z1}, \vec{\kappa}) \right|^2 d^2 \kappa \]  

(4.3.6)

Now, we know that the field Fourier amplitudes are related to the total (coherent plus incoherent) scattering cross-section via:

\[ \sigma^0(\vec{k}_{z1}, \vec{\kappa}) = \frac{4\pi}{A} \left( 2\pi \kappa_z \right)^2 \left| \vec{E}_{\text{scat}}(\vec{k}_{z1}, \vec{\kappa}) \right|^2 \]

And that

\[ \delta(\vec{0}) \approx \frac{1}{(2\pi)^2} \int_A d^2 r = \frac{A}{(2\pi)^2} \]

Which results in the energy conservation relation, valid for all angles of incidence \( \theta_1 \):

\[ \int_{\kappa_{z1}} \frac{\sigma^0(\vec{k}_{z1}, \vec{\kappa})}{4\pi k_{z1} \kappa_z} d^2 \kappa = 1 = \frac{1}{4\pi \cos \theta_1} \left( \int_{2\pi} \sigma^0_{\text{scat}}(\theta_1, \theta, \phi) d\Omega + \int_{2\pi} \sigma^0_{\text{inu}}(\theta_1, \theta, \phi) d\Omega \right) \]  

(4.3.7)

Where \( k_{z1} = k \cos \theta_1 \)  \( \kappa_z = k \cos \theta \)
The scalar energy conservation law (4.3.7), which essentially conveys that all the incident energy becomes either reflected or transmitted through the boundary, can be generalized to vector fields after averaging over input polarizations $p_0$ and summing over output polarizations $p$:

$$
\mathcal{T}_{p_0}(\theta_i) + \sum_p \frac{1}{4\pi \cos \theta_i} \int_{2\pi} \sigma_{p_0}^0 (\theta_i; \theta, \phi) d\Omega = 1 \quad (4.3.8)
$$

Where $\mathcal{T}_{p_0}(\theta_i)$ stands for the hemispherical directional transmittance\(^1\) for incidence angle $\theta$ and input polarization $p_0$, and $\sigma_{p_0}^0$ is the normalized scattering cross-section of the boundary for reflected waves.

4.4 Surface emissivity

Finally, an effort is made to connect the scattering cross-section of the surface to its emissivity, via the concept of hemispherical directional transmittance. The maximum energy which can be radiated by an object in thermal equilibrium at temperature $T$ is called the blackbody radiation. The intensity $I_{\nu}(\nu)$ of the blackbody radiation (defined as power over area over solid angle) emitted in a narrow band of frequencies $\Delta \nu$ is isotropic and given by Planck’s radiation law:

$$
I_0(\nu) = \frac{2h\nu^3}{e^{h\nu/k_B T} - 1} \quad (4.4.1)
$$

Where $h$ is Planck’s constant, $k_B$ is Boltzmann’s constant, $c$ is the speed of light, and $\nu$ is the frequency of the thermal blackbody emission. In the short frequency limit (e.g.

\(^1\) Hemispherical directional transmittance as $\mathcal{T}_{p_0}(\theta_i) = \sum_p \frac{1}{4\pi \cos \theta_i} \int_{2\pi} \sigma_{\text{trans},p_0}^0 (\theta_i; \theta, \phi) d\Omega$
k_B T \gg h \nu, \text{ as for microwave emission from the earth}, \text{ the intensity of radiation emitted by a blackbody appears proportional to the body’s physical temperature:}

$$I_0(\nu) = \frac{2\nu^2}{c^2}k_BT$$

(4.4.2)

The emissivity of an object specifies how well it radiates into a certain direction as compared with a blackbody at the same temperature:

$$\varepsilon(\nu, \theta) = \frac{I(\nu, \theta)}{I_0(\nu)} = \frac{T_b(\nu, \theta)}{T}$$

(4.4.3)

Where $T_b$ is the object’s measured brightness temperature and $T$ its physical temperature. Thus, the emissivity of a surface that encloses a blackbody at equilibrium temperature $T$ is defined as its ability to transmit the blackbody’s internal isotropic radiation (see Figure II.6).

![Figure II.6](image)

Figure II.6 – Surface emissivity: blackbody brightness temperatures before (a) and after (b) insertion of a dielectric discontinuity ($\varepsilon_1$, $\varepsilon_2$).

By reciprocity (i.e. the principle that states that electromagnetic wave solutions in linear media do not vary when the source and the detector are interchanged), the amount of energy $\varepsilon(\theta)$ that is transmitted from all angles below the surface into a direction $\theta$ in the upper medium is equal to the transmittance $\mathcal{T}(\theta)$ for an incidence angle $\theta$ into the lower medium (a.k.a. the hemispherical directional transmittance, see Section 4.3). Namely:
\[ \varepsilon (\theta) = \mathcal{T}(\theta) = -\frac{1}{4\pi \cos \theta} \int_{2\pi}^{\Omega} \sigma_{\text{trans}}^n(\theta; \theta', \phi) d\Omega' \quad (4.4.4) \]

And from the law of energy conservation (see Section 4.3):

\[ \mathcal{T}(\theta) + \frac{1}{4\pi \cos \theta} \int_{2\pi}^{\Omega} \sigma_{\text{scat}}^0(\theta; \theta', \phi) d\Omega' = 1 \quad (4.4.5) \]

Leading to:

\[ \varepsilon (\nu, \theta) = 1 - \frac{1}{4\pi \cos \theta} \int_{2\pi}^{\Omega} \sigma_{\text{scat}}^0(\theta; \theta', \phi) d\Omega' \quad (4.4.6) \]

Where \( \sigma_{\text{scat}}^0 \) is the normalized scattering cross-section of the boundary. In summary, to find a relation between the scattering cross-section and the emissivity of a surface boundary, one needs to obtain the power absorbed by the medium below as the difference between the incident and scattered power, and apply the condition of thermal equilibrium to relate the power absorbed to that which is emitted [Fung, 1994]. This result can be generalized for an inhomogeneous layered medium such as the sea ice cover, as long as the “effective boundary” cross-section accounts for all the absorption, volume and surface scattering effects that occur within the interaction depth.

5 Surface scattering models

5.1 Kirchhoff approximation (KA)

Under the Kirchhoff approximation (KA, a.k.a. physical optics or tangent plane approximation), one assumes that the exact tangential scattered field can be replaced by the wave reflected from an infinite plane locally tangent to the surface at that point. This approximation applies when the typical scale of surface roughness is large compared to the incident wavelength (i.e. \( L >> \lambda \) and \( L >> \sigma \)). Let’s take the incident field to be a monochromatic plane wave with unit amplitude and \( E_h / E_v \) projections onto the (local,
horizontal/vertical) polarization frame, propagating along the unit vector \( \mathbf{n}_{\text{inc}} \) with wavenumber \( k = \omega \sqrt{\varepsilon_0 \mu_0} \), and denote the distance from the surface-to-the-detector as \( R \) (see Appendix F for a description of the reference frame). For (local) horizontally polarized incident electric field, the tangential fields:

\[
\vec{E}_{\text{inc}} = \hat{x} E_{\text{inc}} e^{ik x^0/\varepsilon_0} \\
\vec{H}_{\text{inc}} = \vec{n}_{\text{inc}} \times \vec{E}_{\text{inc}} / Z_0 \\
\vec{E}_{\text{scat}} = \mathcal{R}_h \vec{E}_{\text{inc}} \\
\vec{H}_{\text{scat}} = \mathcal{R}_h \vec{n}_{\text{ref}} \times \vec{E}_{\text{inc}} / Z_0
\]  

(5.1.1)

And for a (local) vertically polarized electric field:

\[
\vec{H}_{\text{inc}} = \hat{x} E_{\text{inc}} / Z_0 e^{ik x^0/\varepsilon_0} \\
\vec{E}_{\text{inc}} = -\vec{n}_{\text{inc}} \times Z_0 \vec{H}_{\text{inc}} \\
\vec{E}_{\text{scat}} = -\mathcal{R}_v \vec{n}_{\text{ref}} \times Z_0 \vec{H}_{\text{inc}} \\
\vec{H}_{\text{scat}} = \mathcal{R}_v \vec{H}_{\text{inc}}
\]  

(5.1.2)

Where \( \mathcal{R}(\beta) \) is the (Fresnel) reflection coefficient for a plane interface between non-permeable media with relative permittivities \( \varepsilon_0 \) and \( \varepsilon \), and \( \beta \) is the incidence angle measured with respect to the local (facet) normal:

\[
\mathcal{R}_h(\beta) = \frac{\cos \beta - \sqrt{\varepsilon - \sin^2 \beta}}{\cos \beta + \sqrt{\varepsilon - \sin^2 \beta}} \\
\mathcal{R}_v(\beta) = \frac{\varepsilon \cos \beta - \sqrt{\varepsilon - \sin^2 \beta}}{\varepsilon \cos \beta + \sqrt{\varepsilon - \sin^2 \beta}}
\]  

(5.1.3)

Substituting the tangent plane approximate fields into the integral equation for the scattered field (4.2.3) gives:

\[
\vec{E}_{\text{scat}} \rightarrow (ik) e^{ikR} \frac{e^{ik R}}{4\pi R} \hat{u} \times \int e^{ikR} \mathcal{R}_h \left( \hat{u} \times (\vec{n}_{\text{ref}} \times \hat{x}) \right) - \left( \vec{n}_{\text{ref}} \times \hat{x} \right) dS
\]

(5.1.4)

Where \( q = \mathbf{k}_{\text{scat}} - \mathbf{k}_{\text{inc}} \) is the scattering vector, and the surface normal is a function of the surface derivatives as:

\[
\vec{n} = \frac{\hat{z} \times \nabla_{\perp} \xi}{\sqrt{1 + 1 \nabla_{\perp} \xi^2}}
\]  

(5.1.5)

In the optical limit (geometric optics, \( k \to \infty \)), the scattered field integral can be evaluated using the asymptotic stationary phase method, such that:

\[
\nabla (\vec{q} \cdot \vec{r})_{\text{sp}} \bigg|_{\text{sp}} = 0 \to \nabla_{\perp} \xi_{\text{sp}} = -\vec{q}_{\perp} / q_z
\]  

(5.1.6)
Implying that the local (facet) normal is parallel to the scattering vector (see Figure II.7):

\[ \hat{n}_{SP} = \frac{\vec{q}}{|\vec{q}|} \]  

(5.1.7)

The key point behind the idea of local specular reflections is that given incidence and scattering directions \( n_{inc}, n_{scat} \), only those surface facets appropriately tilted to beam the signal into the detector contribute to the scattered field, which is thus written:

\[ \tilde{E}_{\text{scat}} \rightarrow (-ik) \frac{e^{iR}}{4\pi R} \hat{C}_{SP} \int e^{i\vec{q} \cdot \vec{dS}} \]  

(5.1.8)

Where the geometric pre-factors \( C_{SP} \) are calculated in Appendix G and given by:

\[ \hat{C}_{SP} = \hat{u}_k \times \left[ \mathcal{R}_h E_h \left( \hat{u}_k \times (\vec{n}_{SP} \times (\vec{n}_{ref} \times \hat{\chi})) - (\vec{n}_{SP} \times \hat{\chi}) \right) \right] \\
+ \mathcal{R}_v E_v \left( \hat{u}_k \times (\vec{n}_{SP} \times \hat{\chi}) + \vec{n}_{SP} \times (\vec{n}_{ref} \times \hat{\chi}) \right) \]  

(5.1.9)

In the scalar (small slope) sub-approximation (i.e. when the difference between local and global angles is neglected):
\[ C_{hh} = 2 \cos \beta \frac{q}{k} \mathcal{R}_h \quad C_{hv} = 0 \]
\[ C_{vv} = 2 \cos \beta \frac{q}{k} \mathcal{R}_v \quad C_{vh} = 0 \] (5.1.10)

Figure II.8 – Source-to-surface (\( \mathbf{R}_1 \)) and surface-to-detector (\( \mathbf{R}_2 \)) vector positions

The phase delay \( k(\mathbf{R}_1 + \mathbf{R}_2) = k(\mathbf{R}_1^0 + \mathbf{R}_2^0) + (\mathbf{r}_{\text{scat}} - \mathbf{k}_{\text{inc}}) \cdot \mathbf{r} + \ldots \)

It is only left to evaluate the integral \( \int e^{i \mathbf{q} \cdot \hat{\mathbf{r}}} dS \) in (5.1.8). The exponent in the integrand is a function of the position vector of surface elements (see Figure II.8):

\[ \hat{\mathbf{r}}(x, y) = x \hat{i} + y \hat{j} + \xi(x, y) \hat{k} \] (5.1.11)

Where \( z = \xi(x, y) \), the surface height function, is a two-dimensional random function that represents the elevation of any given element of surface area from a mean reference level.

If the elements \( dS \) of surface area are required to behave as stationary phase reflectors (from 5.1.6), then the scattering vector \( \mathbf{q} \) must be everywhere parallel to the local normal. Thus:

\[ dS = dx \, dy / \cos(\hat{\mathbf{q}}, \hat{\mathbf{k}}) = \frac{q}{q_z} \, dx \, dy \] (5.1.12)

And the scattering amplitude in the tangent plane approximation:

\[ F(\theta, \phi) = \frac{-ikq}{4\pi q_z} C_{sp} \int e^{i(qz \cdot \xi)} e^{iq \cdot \xi} d^2x \] (5.1.13)
Recall from (3.1) that the scattering cross-section is built from the scattering amplitude as:

\[ \sigma^0(n_{inc}, n_{scat}) = \frac{4\pi}{A} |F(n_{inc}, n_{scat})|^2 \]  

(5.1.14)

Thus we are interested in the statistical average of the total scattering amplitude (coherent + incoherent parts) over the ensemble of \( \xi(x,y) \) realizations:

\[ \langle |F(n_{inc}, n_{scat})|^2 \rangle = \left( \frac{kqC_{sp}}{4\pi q_z} \right)^2 \int \int e^{i\xi_{z} \cdot (\xi - \xi')} < e^{iq_z \cdot (\xi - \xi') >} d^2 x d^2 x' \]  

(5.1.15a)

The coherent part of the scattering amplitude can be expressed as:

\[ \left| |F(n_{inc}, n_{scat})|^2 \right| = \left( \frac{kqC_{sp}}{4\pi q_z} \right)^2 \int \int e^{i\xi_{z} \cdot (\xi - \xi')} < e^{iq_z \cdot \xi} > < e^{-iq_z \cdot \xi'} > d^2 x d^2 x' \]  

(5.1.15b)

Where the average quantities on the right hand side of (5.15a) and (5.15b) are the characteristic functions of the gaussian bivariate and univariate PDF of heights, as introduced in (2.1):

\[ < e^{iq_z \cdot (\xi - \xi') } > = \int \int e^{iq_z \cdot (\xi - \xi') } PDF_h(\xi, \xi') d\xi d\xi' = e^{-q_z^2 \sigma^2 (1 - \rho^2)|\xi - \xi'|} \]  

(5.1.16a)

\[ < e^{iq_z \cdot \xi } > = \int e^{iq_z \cdot \xi } PDF_h(\xi) d\xi = e^{-q_z^2 \sigma^2 / 2} \]  

(5.1.16b)

The total scattering KA cross-section for an isotropic surface:

\[ \sigma^0(n_{inc}, n_{scat}) = 4\pi \left( \frac{kqC_{sp}}{4\pi q_z} \right)^2 \int \int e^{i\xi_{z} \cdot \xi} e^{-q_z^2 \sigma^2 (1 - \rho^2)|\xi - \xi'|} d^2 x \]  

(5.1.17)

And the coherent part of the scattering cross-section:

\[ \sigma_{coh}^0(n_{inc}, n_{scat}) = \pi q_z^2 \Re \left| e^{-q_z^2 \sigma^2 (1 - \rho^2) \delta(\xi)} \right| \]  

(5.1.18)

To proceed further in the evaluation of the total scattering cross-section, one must either determine the explicit form of the surface autocorrelation function, or use a simplifying argument.
Limiting cases

The two classical approaches to rough surface scattering (geometric optics and perturbation theory) arise as limiting cases of the Kirchhoff approximation in (5.1.17), when height deviations are either large or small relative to the effective observation wavelength, here defined in terms of the radiation wavelength, and the incident and scattered wave angles, as:

\[ \lambda_{\text{eff}} = \frac{2\pi}{q_z} = \frac{\lambda}{(\cos \theta_i + \cos \theta_s)} \]  

(5.1.19)

The presence of this parameter in the integral for the cross-section in 5.1.17 (c.f. the Rayleigh criterion) leads to two limiting regimes, namely:

\[ q_z \sigma = \frac{2\pi \sigma}{\lambda_{\text{eff}}} >> 1 \quad \text{large roughness} \]

\[ q_z \sigma = \frac{2\pi \sigma}{\lambda_{\text{eff}}} << 1 \quad \text{small roughness} \]  

(5.1.20)

a) Large roughness limit

In the large-scale roughness limit, \( q_z \sigma >> 1 \), the coherent cross-section vanishes. The characteristic function of the bivariate distribution of surface heights, \( e^{-q_z^2 \sigma^2 (x - x')^{-\rho}}} \), decays rapidly as the separation between surface points increases. The most significant contributions to the cross-section integral in (5.1.17) are expected to come from arguments around \( x = x' \) locations, where we can expand the surface height in Taylor series to arrive at:

\[ < e^{i q_z (\xi - \xi')} > \approx < e^{i q_z (\xi - \xi')} > \approx \int \int e^{i q_z (\xi - \xi')} PDF_{\xi, \phi}(\nabla_{\perp} \xi) d\nabla_{\perp} \xi \]  

(5.1.21)

Under this light, the characteristic function of the bivariate distribution of surface heights becomes a characteristic function of surface gradients [i.e. analogous to (2.6)], and the statistical integral returns the probability density of surface slopes:
The normalized KAGO (Kirchhoff approximation in geometric optics) scattering cross-section in the large roughness limit thus becomes [Barrick, 1968]:

\[ \sigma^0 = \pi \left( \frac{kq}{q_z} \right)^2 \left| C_{sp} \right|^2 \text{PDF}_{\nu_z}(\tilde{q}_\perp) \]  

(5.1.23)

Leading to the same result, an alternative path arises after expanding the correlation function \( \rho \) in (5.1.16a) in a Taylor series about \( x = x' \):

\[ e^{-q_z^2 \sigma^2 |1 - \rho|} \approx e^{-q_z^2 \sigma^2 |1 - \rho| |1 + \frac{1}{2} |\rho| f |^2 f + \ldots} \]  

(5.1.24)

A calculation similar to (5.1.22) provides the actual shape of the probability density of surface slopes for a differentiable random surface (i.e. \( \partial \rho(r)/\partial r \big|_{r=0} \), see Appendix K) with arbitrary autocorrelation function:

\[ \text{PDF}_{\nu_z}(q_x, q_z) = \frac{1}{2\pi mss} e^{-\left(\frac{q_x^2}{q_z^2 mss}\right)} \]  

\[ mss = \left| \frac{\partial^2 \rho(r)}{\partial r^2} \right|_{r=0} = \tan^2 \beta_0 \]  

(5.1.25)

In the limit of large roughness (a.k.a. geometric optics), the scattering cross-section is proportional to the PDF of surface slopes (see Section 2). Observe that the PDF of slopes will produce, in the small slope limit (\( mss \to 0 \)), a specular ray:

\[ \lim_{mss \to 0} \text{PDF}_{\nu_z}(q_x, q_z) = q_z^2 \delta(\tilde{q}_\perp) \quad \rightarrow \quad \sigma^0 = \pi q_z^2 \delta(\tilde{q}_\perp) \]  

(5.1.26)

\[ \text{b) Small roughness limit} \]

In the small roughness limit, \( q_\sigma \ll 1 \), the characteristic function of surface heights can be expanded about its small argument [Beckmann & Spizzichino, 1963]:

\[ e^{\tilde{q}_z \sigma^2 (1 - \rho |\tilde{x} - \tilde{x}'|)} = e^{\tilde{q}_z \sigma^2 \sum_n \left( \frac{q_z^2 \sigma^2 \rho}{n!} \right)n} = e^{\tilde{q}_z \sigma^2 (1 + q_z^2 \sigma^2 \rho(\tilde{x} - \tilde{x}') + \ldots)} \]  

(5.1.27)

Whereby the statistical integral in (5.1.15) becomes approximated as:
\[ \iiint e^{i \mathbf{q} \cdot (\mathbf{x} - \mathbf{x'})} < e^{i \mathbf{q} \cdot (\mathbf{x} - \mathbf{x'})} > d^2 x d^2 x' = A(2\pi)^2 (e^{-i\mathbf{q} \cdot \sigma_i}) \delta(\mathbf{q}_\perp) + q^2 S(\mathbf{q}_\perp) + ... \] (5.1.28)

The normalized scattering cross-section in the small roughness limit becomes:

\[ \sigma^0 = \pi \left( \frac{kq}{q_\perp} \right)^2 \left| C_{SP} \right|^2 e^{-i\mathbf{q} \cdot \sigma_i} \delta(\mathbf{q}_\perp) + \pi (kq)^2 \left| C_{SP} \right|^2 S(\mathbf{q}_\perp) \] (5.1.29)

With a strong coherent ray in the specular direction:

\[ \sigma^0_{coh} = \pi \left( \frac{kq}{q_\perp} \right)^2 \left| C_{SP} \right|^2 \delta(\mathbf{q}_\perp) e^{-i\mathbf{q} \cdot \sigma_i} \] (5.1.30)

And diffuse contributions (obtained at the expense of the specular ray energy) given by:

\[ \sigma^0_{dif} = \pi (kq)^2 \left| C_{SP} \right|^2 S(\mathbf{q}_\perp) \] (5.1.31)

Which is in form identical to the vector cross-section derived later in perturbation theory (only with different geometric coefficients \( C_{SP} \) [Brown, 1978]). In the limit of small roughness, the energy in the scattered field is obtained at the expense of the coherently reflected component, and the scattering cross-section is proportional to the power spectrum of roughness.

\section*{5.2 Small perturbation method (SPM)}

For slightly rough surfaces, an approximate solution to the scattering cross-section is directly calculated using a small perturbation approach (SPM, [Rayleigh, 1945], [Rice, 1951], [Peake, 1959], [Valenzuela, 1968] and [Fuks, 2001]). The idea is to expand the scattered fields as a sum of plane waves propagating upwards (i.e. Rayleigh hypothesis, Appendix H3) as

\[ \hat{p} \cdot \hat{E}_{scat} \bigg|_{z=0} \to \int_{-\infty}^{\infty} d^2 \mathbf{k} \hat{E}_{scat}(\mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{r}} \] (5.2.1)

and determine the unknown plane wave amplitudes \( \hat{E}_{scat}(\mathbf{k}) \) by requiring that the tangential magnetic and electric fields be continuous to first order in surface heights and slopes across the surface. The surface $\xi$ is assumed practically flat, with irregularities less
than a wavelength in depth \( (k\xi << 1) \) and slopes everywhere much less than unity \( (\nabla \perp \xi \ll 1) \). The ‘differential’ vector electromagnetic boundary conditions for non-magnetic media

\[
\hat{n} \times (\vec{E}_{T_2} - \vec{E}_{T_1}) \big|_{z=\xi(x,y)} = 0 \\
\hat{n} \times (\vec{H}_{T_2} - \vec{H}_{T_1}) \big|_{z=\xi(x,y)} = 0
\]

which lead to the appearance of effective electric and magnetic currents on the mean reference plane \( z = 0 \), are made a set of algebraic equations in Fourier space that solve for the scattered field Fourier amplitudes as (see Appendix H):

\[
\vec{E}_{\text{scat},pp_0}(\vec{k}) = c_{pp_0}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}, \epsilon)\vec{\xi}(\vec{q}_\perp)
\]

\( (5.2.2) \)

Where \( c_{pp_0} \) are polarization dependent coefficients and \( p, p_0 = H, V \) (the first index \( p \) denotes polarization of scattered field and the second index \( p_0 \) denotes the polarization of the incident wave). This formula represents the solution of electromagnetic wave diffraction for every random surface \( z = \xi(x,y) \) in the first order of the perturbation theory. Note the resonant (i.e. Bragg) character of scattering: the amplitudes of waves scattered from an incidence direction \( \vec{n}_{\text{inc}} \) into a given direction \( \vec{n}_{\text{scat}} \) are proportional to the Fourier amplitude of roughness with wavenumber \( q_\perp \), the horizontal projection of the scattering vector onto the surface. The scattering amplitude \( F(\vec{n}_{\text{scat}}, \vec{n}_{\text{inc}}) \) is in turn related to the Fourier amplitudes of the scattered field via the relation (3.1.4):

\[
F_{pp_0}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) = -i2\pi\kappa_z \vec{E}_{\text{scat},pp_0}(\vec{k})
\]

\( (5.2.3) \)

The normalized incoherent scattering cross-section \( \sigma^0 \) is thus expressed in terms of the scattering amplitude as (note that in SPM, the coherent field is assumed to be the unperturbed Fresnel reflection from a flat surface):

\[
\sigma^0_{pp_0}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) = \frac{4\pi}{A} \left| \left\langle F_{pp_0}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) \right\rangle \right|^2 = 4\pi \kappa_z^2 \left| c_{pp_0}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) \right|^2 S(\vec{q}_\perp)
\]

\( (5.2.4) \)

Where the expectation brackets indicate average over the ensemble of \( \xi(x,y) \) surfaces, and \( S_\xi \) is the power spectrum of roughness (see Section 2). By substituting the \( c_{pp_0} \) coefficients explicitly, one obtains:
\[ \sigma_{pp}^0(\vec{n}_{inc}, \vec{n}_{sca}) = \pi k^4 |\epsilon - 1|^2 |f_{pp}|^2 S(\vec{q}_L) \] (5.2.5)

Where [Fuks, 2001]

\[
\begin{align*}
    f_{hh} &= (1 + \Re h(\phi_t))(1 + \Re h(\phi_t)) \sin \phi_t \\
    f_{vh} &= -(1 + \Re h(\phi_t))(1 - \Re h(\phi_t)) \cos \phi_t \cos \phi_s \\
    f_{hv} &= -(1 - \Re h(\phi_t))(1 + \Re h(\phi_t)) \cos \phi_t \cos \phi_s \\
    f_{vv} &= (1 + \Re h(\phi_t))(1 + \Re h(\phi_t)) \sin \phi_t \sin \phi_t / \epsilon - (1 - \Re h(\phi_t))(1 - \Re h(\phi_t)) \cos \phi_t \sin \phi_s 
\end{align*}
\]

5.3 Small slope approximation (SSA)

The small slope approximation (SSA) purports to bridge the gap between the scattering regimes delimited by the classical KA and SPM approaches ([Bahar, 1981], [Fung, 1992], [Voronovich, 1994]). For its derivation, postulate a Rayleigh expansion of the scattered field onto plane waves as:

\[ E_s(\vec{r}, z) = \iint_{z=0} \tilde{E}_s(\vec{k}, \vec{k}_s) e^{i(k \cdot \vec{r} + k_z z)} d^2\vec{k} \] (5.3.1)

For a statistically homogeneous random surface, the individual Fourier amplitudes \( \tilde{E}_s(\vec{k}, \vec{k}_s) \) must transform after horizontal and vertical translations of the boundary as (see Figure II.9):

\[ \tilde{E}_s(\vec{k}, \vec{k}_s)\bigg|_{z=h(\vec{r})+d-H} = e^{i(k \cdot \vec{r} - k_z z)} e^{i(k \cdot \vec{r} - k_z z) H} \tilde{E}_s(\vec{k}, \vec{k}_s)\bigg|_{z=h(\vec{r})} \] (5.3.2)

Therefore, a solution for the unknown Fourier amplitudes of the scattered field with already built-in translational properties is sought in the form:

\[ \tilde{E}_s(\vec{k}, \vec{k}_s)\bigg|_{z=h(\vec{r})} = \int \varphi[h(\vec{r}), \vec{r}'; \vec{k}, \vec{k}_s] e^{-i\vec{k}_s \cdot \vec{r}} e^{-i\vec{k} \cdot h(\vec{r})} d^2\vec{r} \] (5.3.3)

Where the functional \( \varphi[h(\vec{r}), \vec{r}] \) should be made by construction invariant to translations of the boundary:

\[ \varphi[h(\vec{r} + \vec{d}), \vec{r} - \vec{d}] = \varphi[h(\vec{r}), \vec{r}] \] (5.3.4)
This is best done in Fourier space:

\[ \varphi[h(\vec{r}), \vec{K}] = \int \Phi[h(\vec{r}), \vec{K}] e^{iK \cdot \vec{r}} \, d^2 K \]  

(5.3.5)

Where the functional \( \Phi[h(r), K] \) will be expanded in an functional Taylor series with argument \( h \) as:

\[ \Phi[h(\vec{r}), \vec{K}] = \Phi_0[\vec{K}] + \int \Phi_1(\vec{K}, \vec{k}) h(\vec{k}) \, d^2 k_1 + ... \]  

(5.3.6)

The translational invariance for the functional \( \varphi[h(r), r] \) is demanded by imposing (5.3.4), with provision for \( h_{r+d}(k) = \exp(ikd) \, h_1(k) \), that is:

\[ \int \Phi[h(\vec{r}), \vec{K}] e^{iK \cdot \vec{r}} \, d^2 K = \int \Phi[h(\vec{r} + \vec{d}), \vec{K}] e^{iK \cdot (\vec{r} + \vec{d})} \, d^2 K \]  

(5.3.7)

With

\[ \int \left( \Phi_0 + \int \Phi_1(\vec{k}) h(\vec{k}) \, d^2 k_1 + ... \right) e^{iK \cdot \vec{r}} \, dK = \int \left( \Phi_0 e^{-ikd} + \int \Phi_1(k_1) h(k_1) e^{i(k_1 - k_1) d} \, dk_1 + ... \right) e^{iK \cdot \vec{r}} \, dK \]

So that the functional \( \Phi[h(r), K] \) can take the form:

\[ \Phi[h(\vec{r}), \vec{K}] = \tilde{\Phi}_0 \delta(K) + \int \tilde{\Phi}_1(k_1) h(k_1) \delta(K - k_1) \, dk_1 + ... \]  

(5.3.8)

And the functional \( \varphi[h(r), r] \) be written as:

\[ \varphi(h, r) = \tilde{\Phi}_0 + \int \tilde{\Phi}_1(k_1) h(k_1) e^{ik_1 r} \, dk_1 + ... \]  

(5.3.9)

Then the scattered field Fourier amplitudes in the first order SSA become:
\[ \tilde{E}_i(\vec{k}, \vec{k}) = \int \Phi_0 e^{-iq_k \cdot \delta} e^{-iq_b \cdot \delta'} d^2 r + O(h) \] (5.3.10)

Where the discarded \( O(h) \) term is proportional to the surface slope [Voronovich, 1994].

To determine the SSA cross-section to first order, let the solution approach asymptotically the \( h \to 0 \) limit and match the result to the SPM solution in (5.2.2):

\[ \int \Phi_0 e^{iq_k \cdot \delta} e^{iq_b \cdot \delta'} d^2 r \to \int \Phi_0 e^{iq_k \cdot \delta} (1 + iq_k \cdot h(r))d^2 r = (2\pi)^2 \Phi_0 \left( \delta(q_\perp) + iq_k \tilde{h}(q_\perp) \right) \]

A straightforward comparison yields:

\[ \Phi_0 = \frac{k^2}{2(2\pi)^2 k_z q_z} (\varepsilon - 1) f_{pp} \] (5.3.11)

Therefore, the first order small slope approximation of the scattered field (mean plus fluctuation) is built:

\[ \tilde{E}_i(\vec{k}, \vec{k}) = \frac{k^2}{2k_z q_z} (\varepsilon - 1) f_{pp} \left\{ \int e^{iq_k \cdot \delta} e^{iq_b \cdot \delta'} d^2 r \right\} \] (5.3.12)

The normalized total (coherent plus incoherent) scattering cross-section is formed using (3.1.5):

\[ \sigma_{pp}^0 (\vec{n}_{inc}, \vec{n}_{scat}) = \pi \left( \frac{k^2}{2\pi q_z} \right)^2 (\varepsilon - 1) f_{pp} \left\{ \int e^{iq_k \cdot \delta} e^{iq_b \cdot \delta'} d^2 r \right\} \] (5.3.14)

Which is valid for random gaussian surfaces with arbitrary autocorrelation. Observe that the SPM and the KA solutions in (5.2.5) and (5.1.17) merge in the SSA cross-section (5.3.14) above, which bears the geometric pre-factors from the former and the statistical average from the latter. It can be proved that the small slope cross-section approaches the KA solution for near-specular angles \( q_\perp \sim 0 \) and the SPM solution for small surface deviations. In practical calculations, it is convenient (i.e. more stable numerically) to calculate the incoherent cross-section first and add the coherent component later. For the coherent portion of the field:

\[ \left\{ \tilde{E}_i(\vec{k}, \vec{k}) \right\} = \frac{k^2}{2k_z q_z} (\varepsilon - 1) f_{pp} \left\{ \int e^{iq_k \cdot \delta} \left\{ e^{iq_b \cdot \delta'} \right\} d^2 r \right\} \] (5.3.15)

Where the statistical average is given by (5.1.16b) to yield the specular reflectivity:
\[ \mathcal{E}_i(\mathbf{k}, \mathbf{\tilde{k}}) = \mathcal{R}_{\text{eff}} \delta(\mathbf{\tilde{q}}) = \frac{k^2}{2\kappa q_z} (e - 1) f_{pp} e^{-q_z \sigma^2/2} \delta(\mathbf{\tilde{q}}) \]  

(5.3.16)

5.4 Coherent scattering

From the radar equation (see Section 3.1 in Chapter I), the power intercepted by an appropriately oriented detector can be written as (ignore here the GPS signal autocorrelation properties):

\[ P = |E_{\text{inc}}|^2 A_d \int_{\text{surface}} \frac{\sigma^0}{4\pi R^2} d^2 r \]

Where \(|E_{\text{inc}}|^2\) is the incident power flux density (= \(P_G/4\pi R_i^2\)) and \(A_d\) is the effective aperture of the detector (= \(\lambda^2 G_r/4\pi\)). The detected power, which first appears as an integral over intensities, becomes an integral over amplitudes when a delta function is present in the total cross-section. For a perfectly flat surface [from (5.1.18) or (5.3.16)]:

\[ \sigma^0 = \pi q_z^2 |\mathcal{R}_{\text{eff}}|^2 \delta(\mathbf{\tilde{q}}) \]  

(5.4.1)

\[ P \longrightarrow |\mathcal{R}_{\text{eff}} E_{\text{inc}}|^2 A_d \int_{\text{surface}} \frac{\delta(\theta - \theta_i)\delta(\phi - \phi_i)}{\sin \theta_i} d\Omega = |\mathcal{R}_{\text{eff}} E_{\text{inc}}|^2 A_d \]

Where \(d\Omega = \cos \theta_i \frac{d^2 r}{R^2}\) and \(\delta(\mathbf{\tilde{q}}) = \frac{\delta(\theta - \theta_i)\delta(\phi - \phi_i)}{k^2 \sin \theta_i \cos \theta_i}\)  

(5.4.2)

And the coherently detected power becomes \(P = |\mathcal{R}_{\text{eff}} E_{\text{inc}}|^2 A_d\). Conversely, if one takes the specularly reflected ray and writes it as an expansion in Fourier components (using Rayleigh hypothesis, see Appendix H3):

\[ E_{\text{coh}} = \mathcal{R}_{\text{eff}} e^{i \mathbf{\tilde{k}} \cdot \mathbf{r}} = \int \mathcal{E}_{\text{coh}}(\mathbf{\tilde{k}}) e^{i \mathbf{\tilde{k}} \cdot \mathbf{r}} d^2 \mathbf{\tilde{k}} \]  

(5.4.3)

With \(\mathcal{E}_{\text{coh}}(\mathbf{\tilde{k}}) = \mathcal{R}_{\text{eff}} \delta(\mathbf{\tilde{k}} - \mathbf{\tilde{k}}_{\text{ref}})\)

Then the cross-section of the specular ray becomes, via (3.1.5):

\[ \sigma^0_{\text{coh}} = \frac{4\pi}{A} (2\pi \kappa_z)^2 |\mathcal{E}_{\text{coh}}(\mathbf{\tilde{k}})|^2 = \ldots = \frac{4\pi}{A} (2\pi \kappa_z)^2 |\mathcal{R}_{\text{eff}}|^2 \left[ \delta(\mathbf{\tilde{k}} - \mathbf{\tilde{k}}_{\text{ref}}) \right]^2 \]  

(5.4.4)
Where \( \delta(\vec{k}_\perp - \vec{k}_{ref \perp}) = \delta(\vec{q}_\perp) \) and therefore \( \kappa_z = q_z / 2 \)

A possible way to interpret the squared delta function that appears in the coherent cross-section:

\[
\delta(\vec{q}_\perp) \approx \frac{1}{(2\pi)^2} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} e^{i\vec{q}_\perp \cdot \vec{r}} d^2 r = \begin{cases} \frac{A}{(2\pi)^2} & \text{if } q_\perp = 0 \\ 0 & \text{otherwise} \end{cases}
\]

(5.4.5)

So that the coherent cross-section of the specular ray would finally appear as:

\[
\sigma^0_{coh} = \frac{4\pi}{A} (2\pi\kappa_z)^2 |\mathcal{R}_{eff}|^2 |\delta(\vec{k}_\perp - \vec{k}_{ref \perp})| \frac{A}{(2\pi)^2} = \pi q_z^2 |\mathcal{R}_{eff}|^2 \delta(\vec{q}_\perp)
\]

(5.4.6)

In agreement with (5.1.18) for KAGO and (5.3.16) for SSA with

\[
\mathcal{R}_{eff} = \Re\frac{(q,\sigma)^2}{2}
\]

(5.4.7)

Therefore, the specularly reflected ray admits two equivalent representations: one as a plane wave with an effective coefficient of reflection \( \mathcal{R}_{eff} \), or as the coherent part of the total scattering cross-section \( \sigma^0_{coh} \).

6 Model domains

6.1 KA/SPM

From the set of assumptions made in deriving the respective solutions, the Kirchhoff approximation will give the correct result in the optical limit (\( \lambda \rightarrow 0 \)) when the roughness parameters \( (\sigma_l, L_l) \) of a gaussian correlated surface satisfy:

\[
L_l >> \lambda_{eff} \quad \text{and} \quad L_l >> \sigma_l \quad \text{small surface curvature and}
\]

\[
2\pi\sigma_l/\lambda_{eff} >> 1 \quad \text{large roughness limit (KA and KAGO) or}
\]

\[
2\pi\sigma_l/\lambda_{eff} << 1 \quad \text{small roughness limit (KA)}
\]

(6.1.1)
Where the effective observation wavelength depends on the radiation wavelength and the incidence and scattered angles, as defined in (5.1.19), from $\lambda_{eff} = \lambda/2$ for nadir backscatter to $\lambda_{eff} = 3\lambda$ for backscatter at 80 degrees from nadir. Note that these conditions are in the form of approximate inequalities, which indicate that there are no exact boundaries for the regions of applicability.

The small perturbation method in turn requires that surface irregularities be much less than a wavelength in depth and slopes everywhere much less than unity. This in terms of the roughness parameters ($\sigma_s, L_s$) of a gaussian correlated surface as [Bass & Fuks, 1979]:

\[ L_s << \frac{\lambda_{eff}}{2\pi} \rightarrow 2\pi\sigma_s/L_s << 1 \quad \text{small scale limit or} \]

\[ L_s >> \frac{\lambda_{eff}}{2\pi} \rightarrow 2\pi\sigma_s/\lambda_{eff} << 1 \quad \text{large scale limit} \quad (6.1.2) \]

Figure II.10 – Spectral domains of validity of the classical KA and SPM models for isotropic gaussian correlated surfaces

Note that the first condition in (6.1.2) for SPM in the large scale limit is equivalent to second condition in (6.1.1) for KA in the small roughness limit, which results in partially overlapping domains of applicability. The validity of the first order KA and SPM models for various parameters ($\sigma$, $L$) of a Gaussian correlated and perfectly conducting surface
have been examined using numerical Monte-Carlo methods by different authors ([Chen & Fung, 1988], [Thorsos, 1988] and [Thorsos & Jackson, 1989]), providing further evidence for the conditions above and a new piece of information, namely, that both KA and SPM models must be evaluated to higher order in the region where their validity overlaps for their satisfactory agreement. At low grazing angles, the validity of the classical models shows an additional degradation associated with shadowing and multiple scattering [Thorsos, 1988]. This degradation is particularly severe for the Kirchhoff approximation, since the condition $L >> \lambda_{\text{eff}}$ is violated as $\lambda_{\text{eff}}$ increases with incidence and scatter angles.

Figure II.10 shows the approximate spectral domains of validity of the KA/SPM scattering models for gaussian correlated surfaces. The roughness spectra $S(k; \sigma, L)$ in the figure are taken from (2.8) and fed with parameters $\sigma_i, L_i$ complying with (6.1.1) for the KA model, and $\sigma_s, L_s$ complying with (6.1.2) for the SPM model. The Kirchhoff approximation typically describes strong near-specular scattering from large-amplitude large-scale roughness components ($\sigma_i, L_i >> \lambda_{\text{eff}}$), while the small perturbation method typically covers the regime of weak Bragg/resonant scattering from small-amplitude small-scale roughness components ($\sigma_s, L_s << \lambda_{\text{eff}}$) at near and further off-specular angles. An important remark is that, by construction, neither KA nor the SPM are sensitive to roughness with wavenumbers $k > 2k_0$, where $k_0 = 2\pi/\lambda$, implying that the performance of these first order models will deteriorate when significant amounts of roughness are present at scales smaller than a half-wavelength.

In general, roughness spectra of natural surfaces do not fall entirely in the domain of validity of either classical model. The KA works well in the forward scatter directions (i.e. for small $q_\perp$) but gives inaccurate results in directions far from specular, when the roughness spectrum is broad and a contribution from Bragg scattering (i.e. SPM) cannot
be neglected [Zavorotny & Voronovich, 1999]. Alternative strategies have been
developed for these cases: the two scale model and the small slope approach.

6.2 Two-scale model

Composite models have been developed in a rather heuristic manner to explain radar
returns from the sea surface in a backscatter geometry ([Wright, 1968], [Wu & Fung,
1972], [Chan & Fung, 1973] and [Wentz, 1977]). The two-scale surface is modeled as
having two statistically independent components: a small-amplitude high-frequency
component (with \( L_s < \lambda_{\text{eff}} \) and \( \sigma_{s,k_{\text{eff}}} < 1 \), described by SPM) superimposed on a low-
frequency component of larger amplitude (with \( L > \lambda_{\text{eff}} \) and \( \sigma_{L,k_{\text{eff}}} > 1 \), described by
KAGO). The resulting two-scale cross-section:

\[
\sigma^0(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) = \sigma^0|_{\text{KAGO}} + \left\{ \sigma^0|_{\text{SPM}} \right\}_{\text{tilts}} \tag{6.2.1}
\]

With

\[
\sigma^0(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}})|_{\text{KAGO}} = \pi \left( \frac{kq}{q_z} \right)^2 \left| C_{SP} \right|^2 \left[ \frac{1}{2} \int_{-\infty}^{\infty} \left| f_{pph} \left( \vec{q}_\perp \right) \right| \right] \]

\[
\sigma^0(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}})|_{\text{SPM}} = \pi k^4 \left| \epsilon - 1 \right|^2 \left| f_{pph} \right|^2 S(\vec{q}_\perp) \]

Where the effects of small-scale roughness are corrected by averaging over the large-
scale surface tilts (i.e. ‘tilted perturbation method’, see Appendix I). A complete
parameterization of a two-scale surface thus requires specification of a large-scale mean
square slope for the KAGO part and the small-scale roughness power spectrum for the
SPM part. A plausible construction of a two-scale model would require that the actual
parameters that go into the KAGO/SPM models correspond to low- and high-pass filtered
versions of the surface, with a choice of filter cutoff \( k_{\text{cutoff}} \) that ensures that both the
KAGO and SPM conditions are satisfied ([Brown, 1978], [Brown, 1990]), but that
nevertheless remains ambiguous and subject to discussion ([Barrick & Peake, 1968],

6.3 SSA

Encompassing both the Kirchhoff and resonant scattering mechanisms, the SSA is applicable irrespective of the roughness scales involved, as long as their slopes are small compared with the angles of incidence and scattering [Voronovich, 1994]. Using a gaussian correlated and perfectly conducting surface, the first order SSA is found to be generally accurate, at least to the extent that its asymptotic limit, the first order SPM, remains accurate [Soriano et al., 2002]. Addition of higher order terms in the SSA cross-section series yields progressively better results, even beyond the region where higher order SPM is accurate (Thorsos & Broschat, 1995], [Broschat & Thorsos, 1997]).

7 Numerical simulations

Finally, we proceed to calculate the angular distribution of scattered radiation for particular rough surfaces using the scattering models just introduced: KAGO (5.1.23), KA (5.1.17), SPM (5.2.5) and SSA (5.3.14). We consider a typical sea ice interface having relative dielectric permittivity $\varepsilon = 4$ and modeled as a random gaussian variable with expo-gaussian autocorrelation (see Section 2):

$$\rho(r; \sigma, L_1, L_2) = \sigma^2 e^{-r^2/(2L_1^2)} = \begin{cases} \sigma^2 e^{-r^2/(2L_1^2)} & r << L_2 \\ \sigma^2 e^{-r^2/L_1^2} & r >> L_2 \end{cases}$$  (7.1)

This random surface model adjusts to measurements of large scale roughness of sea ice provided by an airborne LIDAR profiler (exponential $L_1$ parameter, see Section 2), while it allows studying the effect of varying degrees of small scale roughness in model cross-sections (spectral dampening $L_2$ parameter, see Figure II.11). It also overcomes the
The observing wavelength $\lambda_{\text{GPS}} = 0.19$ m ($k_{0,\text{GPS}} = 33.1$ rad/m) and the large scale exponential correlation length is chosen $L_1 = 5.5$ m. The scale of transition from exponential to gaussian correlated surfaces, $L_2$, varies between 0.5 m (for a mostly exponential surface, with significant small scale roughness) to 5 m (for a mostly gaussian surface). In the following figures, in-plane scattering cross-section patterns for different incidence angles and LR output/input polarization are shown (see Appendix J for notes on how to effect the change in polarization basis from HV to LR). These figures are intended to illustrate a few interesting points about approximate solutions to the scattering cross-section from a random surface, namely:

i) For small surface heights (smooth surfaces, in Rayleigh sense), all models (KA, SPM, SSA) converge to the same solution at near specular angles. The KA cross-section, which only differs from the KAGO cross-section in that
the latter adopts a quadratic approximation to the surface autocorrelation, only deviates from the SSA and SPM solutions at low grazing scatter angles (see Figure II.12-A).

ii) For increasing surface heights (rough surfaces, in Rayleigh sense), the SPM solution diverges at near specular angles, but remains accurate at low grazing angles. The KA solution breaks down at low grazing scatter angles, but remains accurate at near specular angles. The SSA model provides a solution of continuity between the KA and SPM valid domains of representation (see Figure II.12-B).

iii) For surfaces devoid of small scale roughness (i.e. gaussian correlated, $L \gg \lambda$), SPM gives null resonant/Bragg scattering, and the rest of the models (KA, KAGO and SSA) converge to the same solution (see Figure II.13). That is, in absence of resonant/Bragg scatter arising from small scale roughness, the surface cross-section admits a simple and accurate KAGO representation.

iv) As the level of small scale roughness increases (in passing from a gaussian correlated surface to an exponentially correlated surface, see Figures II.14 and II.15), resonant/Bragg contributions lift the cross-section tails at off-specular angles and near-specular scattering abandons its quadratic form, as higher order terms in the series expansion of the autocorrelation become necessary. The KAGO representation, which absorbs the small scale slopes as if they were large scale features (by virtue of the quadratic approximation, see Section 2), becomes less and less accurate. In practice, this effect can be partly corrected for by imposing a wavenumber cutoff in the KAGO mean square slope that minimizes the difference between KAGO and KA/SSA cross-sections at near specular angles (see Section 6.2). This cutoff
wavenumber generally lies about $k_0$, but depends on the roughness spectrum and the observation geometry (and thus can only be used to make brute inferences about the correlation length of the large scale structure of the surface). For surfaces with a significant degree of small scale roughness, the KAGO model with an adjusted cutoff wavenumber remains a low order approximation to specular scattering, while SSA retains the full shape of the surface spectrum across scatter angles.

The overall level of the KA/SSA cross-sections increases in passing from the finite dielectric to the perfectly conducting interface cases, although the shape of the cross-section is only slightly modified (see Figure II.16). The glistening zone is defined as the region on the surface that redirects a significant amount of power towards the receiver, and by significant is meant detectable or above the noise floor of the detector. For scattered GPS waveforms (see Section 3.3 in Chapter I), one can establish an upper bound to the power arising from the surface as:

$$\chi_{\text{scat}}(\tau) = \int_{-\infty}^{\infty} \frac{\sigma^0(r) \chi(\tau, \gamma)}{4\pi R^2(r)} d^2r \approx \frac{\sigma^0 A_{\text{chip}}}{4\pi R^2} \approx \sigma^0/h$$

Where $A_{\text{chip}} = 4\pi h / \sin^2 \gamma$, $R = h / \sin \gamma$, $h$ is the receiver altitude and $\gamma$ the signal elevation angle. For typical detector noise floor levels of -20 dB and low flight altitudes ($h \sim 1$, in TCA units), the glistening zone will be limited to sectors with normalized cross-sections in excess of -20 dB. This detection limit is indicated in Figures II.12 through II.16 below with a dashed line, highlighting the features in model cross-sections that will eventually become observable experimentally.
A) Smooth surface $\sigma = 0.01$ m

B) Slightly rough surface $\sigma = 0.08$ m

Figure II.12 – Comparison of SPM/KA/SSA incoherent cross-sections for an exponential surface ($L_1 = 5.5$ m)
A) Slightly rough surface $\sigma = 0.08$ m

B) Rough surface $\sigma = 0.25$ m

Figure II.13 – Comparison of KAGO/KA/SSA incoherent cross-sections for a gaussian surface ($L_1 = 5.5$ m)
Figure II.14 – Comparison of KA/SSA incoherent cross-sections for an expo-gaussian surface ($L_1 = 5.5$ m, $L_2 = 5$ m)

A) Slightly rough surface $\sigma = 0.08$ m

B) Rough surface $\sigma = 0.25$ m
Figure II.15 – Comparison of KA/SSA incoherent cross-sections for an expo-gaussian surface ($L_1 = 5.5$ m, $L_2 = 0.5$ m)

A) Slightly rough surface $\sigma = 0.08$ m

B) Rough surface $\sigma = 0.25$ m
A) Slightly rough surface $\sigma = 0.08$ m

B) Rough surface $\sigma = 0.25$ m

Figure II.16 – Comparison of KA/SSA incoherent cross-sections for an expo-gaussian surface ($L_1 = 5.5$ m, $L_2 = 0.5$ m) for finite dielectric ($\varepsilon = 4$, lower curves) and perfectly conducting ($\varepsilon = 100000$, upper curves) interfaces
Energy conservation

The law of conservation of energy provides a reliable means to verify the domain of applicability of a scattering model (see Section 4.3) and it is essential to studies of surface emission. The relation states that for a plane wave hitting an interface from an incident angle $\theta_i$:

$$\frac{1}{4\pi \cos \theta_i} \left( \int_{\Omega_{\text{up}}} \sigma_{\text{scat}}^0 (\theta_i; \theta, \phi) d\Omega + \int_{\Omega_{\text{down}}} \sigma_{\text{trans}}^0 (\theta_i; \theta, \phi) d\Omega \right) = 1$$  \hspace{1cm} (7.3)

The relation simplifies when applied to a perfectly conducting boundary (i.e. $\sigma_{\text{trans}}^0 = 0$).

For incoming RHCP radiation hitting a perfectly conducting surface, the total (coherent and incoherent) scattering cross-sections must obey:

$$\frac{1}{4\pi \cos \theta_i} \int_{\Omega_{\text{up}}} (\sigma_{\text{LR}}^0 (\theta_i; \theta, \phi) + \sigma_{\text{RR}}^0 (\theta_i; \theta, \phi)) d\Omega = 1$$  \hspace{1cm} (7.4)

We will compute the energy balance that results from the KAGO and SSA cross-sections for a random gaussian surface with an expo-gaussian autocorrelation function such as (7.1), with typical surface parameters $L_1 = 5.5$ m and $L_2 = 0.5$ m. The surface RMS heights range from 0.01 to 0.50 m, corresponding to average surface slopes of 0.5 to 25 degrees. The amount of energy in the coherent part of the cross-section (specular reflectivity, see Section 5.4) is model independent and shown in Figure II.17 below.

![Coherent energy](image)

**Figure II.17 – Coherent reflected energy**
Figures II.18 and II.19 show that both KAGO and SSA models comply acceptably well with the energy conservation relation for a perfectly conducting interface for all incidence angles under about 65 degrees. The range of incidence angles for which the KAGO and SSA models can be considered acceptable decreases as the average surface slope increases and the departure towards energy overestimation at large incidence angles is related to unaccounted shadowing effects [Tsang et al., 1985]. Although not shown in the figures, the amount of energy that is scattered into the cross-polarized RR mode according to SSA is about 1-5% that which is reflected into the LR mode (i.e. between 13
and 20 dB below the LR mode, increasing with surface roughness and incidence angle), in contrast to the KAGO model, which predicts null RR scatter. The degree of signal depolarization predicted by SSA is in qualitative good agreement with experimental measurements of LHCP and RHCP GPS reflections from the sea surface [Thompson et al., 2003].

**Polarization structure**

In the KAGO model, slope terms in the local coordinate vectors are neglected and polarization changes at local facets ignored, thereby suppressing the cross-polarized components of the scattered field (i.e. $\sigma_{vh}^0 = \sigma_{hv}^0 = \sigma_{RR}^0 = 0$). The polarization structure of scattered radiation in the SSA model (same as for the SPM model) is shown in Figure II.20 and Figure II.21 below for a perfectly conducting surface, for nadir incidence and initial linear RHCP and V-pol excitations.

![Figure II.20 – SSA/SPM polarization structure for nadir incidence and initial RHCP-pol](image-url)
From Figure II.20, the angular patterns of LR and RR radiation for an RHCP signal incident on a rough surface are isotropic with respect to the azimuthal angle of scatter. Observe that RR radiation is to be observed mainly at low grazing scatter angles, according to the SSA model. Figure II.21 for a linearly polarized signal incident on a rough surface reproduces the angular pattern of a surface dipole oriented parallel to the incident field.

**8 Summary**

The propagation of an electromagnetic disturbance in a physical medium is a well defined entity ruled by Maxwell’s equations. From the requirement for continuity of the
tangential field components across boundaries, an electromagnetic disturbance that
encounters a discontinuity in electric properties will give rise to scattered components.

The surface scattering models developed in this chapter are single scattering (first order)
solutions that neglect multiple scattering and shadowing effects, and can be used to study
the bistatic cross-section and emissivity of a rough surface. To a first order
approximation, the angular pattern of scattering from a statistically rough surface depends
on the magnitude of the dielectric discontinuity at the boundary and the autocorrelation
function of surface deviations. In the limit of small surface deviations, the scattering
cross-section has most of the reflected power confined to the specular direction, with
weak scattering sidelobes proportional to the surface spectrum (SPM). In the limit of
large surface deviations, the scattering cross-section becomes proportional to the PDF of
surface slopes, provided that the surface remains essentially flat at scales smaller than the
effective radiation wavelength (KAGO). In the limit of small slopes, the scattering cross-
section becomes proportional to the Fourier transform of the characteristic function of the
joint PDF of surface heights (SSA), approaching asymptotically the two former large and
small roughness limits at near and far off-specular angles respectively.

Both the KAGO and SSA models are applicable to the study of forward scattering of L-
band signals from sea ice. The KAGO model is very amenable due to its simplicity (since
it describes the surface cross-section using a single statistical parameter, the surface mean
square slope), but it imposes severe constraints on the scales of roughness that contribute
to scattering under this limit, with ensuing difficulties in establishing meaningful PDF of
slopes for surfaces that do not fall entirely within its domain of validity [Ulaby et al.,
1986]. The SSA model remains valid for a wider range of surface types. However, its
implementation is more involved, since it requires a complete statistical description of the
random surface (the full autocorrelation function) in the calculation of the cross-section.
CHAPTER II – Results
1 Introduction

In this chapter, we apply the models developed previously to the inversion of surface parameters using scattered GPS waveforms collected from an airborne platform. For the validation of the model inversion results, reference surface parameters such as roughness and sea ice type are first generated from independent but time and space co-located sensors.

2 Data collection

In our GPS bistatic experiment, the sources of transmission of L-Band probing signals are the satellites of the GPS constellation. The GPS radar receiver is located on a NASA P3 aircraft, a large payload carrier that is host to the GPS radar system (GPS/RS in Figure III.1 below), as well as to a conically scanning LIDAR profiler (Airborne Topographic Mapper, ATM) and a conically scanning microwave radiometer (Polarimetric Scanning Radiometer, PSR). The airborne payload is complemented with ground GPS reference stations for precise data geo-registration and SAR (Synthetic Aperture Radar) imagery from the RADARSAT satellite.

![NASA P3 platform: location of sensors onboard](image1)

Figure III.1 – NASA P3 platform: location of sensors onboard
The data collection was executed over the Arctic in the month of March 2003 under the AMSRIce03 campaign for the validation of the AMSR sensor (Advanced Microwave Scanning Radiometer, [Cavalieri et al., 2006]) aboard the NASA-EOS satellite AQUA. The campaign comprised a number of flights over typical arctic spring scenarios over the Beaufort, Bering and Chukchi Seas (see Figure III.2 and Table III.1 below).

![Figure III.2 – AMSRIce03 flight targets](image)

<table>
<thead>
<tr>
<th>Flight</th>
<th>Date (mm/dd)</th>
<th>General sea ice conditions</th>
<th>Altitude (m)</th>
<th>PSR data</th>
<th>GPS data</th>
<th>LIDAR data</th>
<th>SAR data</th>
</tr>
</thead>
<tbody>
<tr>
<td>A – Point Barrow</td>
<td>03/13</td>
<td>Shorefast ice</td>
<td>150,1200</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>B – Icecamp</td>
<td>03/19</td>
<td>Multiyear ice</td>
<td>1200</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>C – St Matthew</td>
<td>03/18</td>
<td>Sea ice edge</td>
<td>1200</td>
<td>√</td>
<td>√</td>
<td>NO</td>
<td>√</td>
</tr>
<tr>
<td>D – Norton Sound</td>
<td>03/15</td>
<td>Thin ice</td>
<td>1200</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>E – St Lawrence</td>
<td>03/16</td>
<td>New ice</td>
<td>150</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>F – Point Hope</td>
<td>03/20</td>
<td>Thin ice</td>
<td>150,1200</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>G – Bering Sea</td>
<td>03/22</td>
<td>Sea ice edge</td>
<td>1200</td>
<td>√</td>
<td>√</td>
<td>NO</td>
<td>√</td>
</tr>
</tbody>
</table>

2.1 Data collocation

Combining observations from a variety of satellite and airborne sensors results in a mixture of coverage, collocation and spatial resolution issues. Spatial collocation is
achieved by georeferencing all data onto a Universal Transversal Mercator grid (UTM, zone 4). Further details about the observation geometries and resolution of the different sensors are provided next.

2.1.1 Polarimetric radiometer

The Polarimetric Scanning Radiometer (PSR) is an airborne conically scanning imaging radiometer. The PSR antennas provide full polarimetric sensitivity at 10.7, 18.7 and 37.0 GHz at a scanning incidence angle of 55 degrees from nadir [Piepmeier & Gasiewski, 1996]. The pixel size is determined by the PSR antenna beamwidth (see Table III.2). The PSR microwave brightness temperatures are complemented by a KT19 infrared radiometer, which measures radiation in the infrared atmospheric window at 9.6 - 11.5 µm, for the estimation of microwave emissivities via surface skin temperatures.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Beamwidth</th>
<th>Footprint diameter</th>
<th>Swath</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.7 GHz (H and V)</td>
<td>8 deg</td>
<td>500 m</td>
<td></td>
</tr>
<tr>
<td>18.7 GHz (H and V)</td>
<td>8 deg</td>
<td>500 m</td>
<td></td>
</tr>
<tr>
<td>21.5 GHz (H and V)</td>
<td>8 deg</td>
<td>500 m</td>
<td></td>
</tr>
<tr>
<td>37.0 GHz (H and V)</td>
<td>2.3 deg</td>
<td>150 m</td>
<td>1700 m</td>
</tr>
<tr>
<td>KT19 infrared</td>
<td>7 deg</td>
<td>450 m</td>
<td></td>
</tr>
</tbody>
</table>

2.1.2 Synthetic aperture radar

The Alaska SAR Facility archive of RADARSAT-1 ScanSAR Wide data has provided the set of synthetic aperture radar images, with pixel resolution of 100 m. The RADARSAT instrument operates at the C-Band (5.3 GHz) receiving only HH backscatter and covering local incidence angles that range from 20 to 50 degrees [Leung et al., 1996]. The SAR images are calibrated radiometrically, both relatively for antenna beam pattern and space loss, and absolutely using distributed targets of known radar cross-section.
2.1.3 LIDAR profiler

The ATM is a conically scanning LIDAR mounted on the NASA P3 aircraft [Krabill et al., 2002]. The ATM LIDAR profiler measures the round-trip travel time of a laser pulse between the aircraft and the surface and combines this information with precise dual-frequency kinematic GPS and an onboard inertial system to obtain surface height profiles referenced to the WGS-84 ellipsoid (Figure III.3 and Table III.3).

Figure III.3 – ATM observation geometry and parameter definition (see Table below)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser wavelength</td>
<td>523 nm</td>
</tr>
<tr>
<td>Laser pulse rate</td>
<td>5000 pps</td>
</tr>
<tr>
<td>Scanner rotation rate $R_{\text{scan}}$</td>
<td>20 Hz</td>
</tr>
<tr>
<td>Scan sampling</td>
<td>250 pulses/scan</td>
</tr>
<tr>
<td>Scanner off-nadir angle</td>
<td>15 degrees</td>
</tr>
<tr>
<td>Flight altitude $h$</td>
<td>150 – 1200 m</td>
</tr>
<tr>
<td>Swath width $W$</td>
<td>75 – 600 m</td>
</tr>
<tr>
<td>Laser beam divergence</td>
<td>0.0025 rads</td>
</tr>
<tr>
<td>Laser footprint diameter $D$</td>
<td>0.25 – 3 m</td>
</tr>
<tr>
<td>Along-scan sample distance $\Delta x$</td>
<td>1 – 8 m</td>
</tr>
<tr>
<td>Across-scan sample distance $D$</td>
<td>~ 10 m</td>
</tr>
<tr>
<td>Vertical Accuracy $D$</td>
<td>~ 0.1 m</td>
</tr>
</tbody>
</table>

(1) Depends on flight altitude. (2) Depends on quality of GPS kinematic processing.
2.1.4 GPS bistatic radar

Scattered GPS signals are processed using a 12-channel software configurable GPS receiver [Mitel, 1999] connected to both down-looking LHCP (left hand polarized) and up-looking RHCP (right hand polarized) hemispherical L-band antennas, each having independent AGCs (automatic gain controls). The configurable receiver consists of an array of 14 correlators with code replicas spaced 1/2 CA code chips apart, which accumulate 1 msec power samples using the Doppler frequency of the direct signal (see Figure III.4). The down-looking correlators are slaved to the up-looking channel via a surface topography model for optimum waveform tracking. The correlation waveforms are incoherently accumulated over periods of 1 second to reduce signal speckle, an averaging period over which the aircraft altitude and the look angle are assumed constant.

Figure III.4 – Diagram of GPS bistatic correlation receiver (from [Garrison et al., 2002])

Figure III.5 – Data collocation in ASMRIce03 flights
For these flights, the GPS bistatic radar keeps track of one single scattered waveform at a time, with a satellite elevation lying between 25° and 65°. The GPS reflection ground-tracks are geo-referenced using IGS (International Geodetic Service) precision GPS orbits and aircraft route information (from a conventional GPS receiver), after terrain correction based on a 1-km resolution digital elevation model (GTOPO30). The size of the GPS bistatic footprint is determined by the GPS pseudo-random code length, $T_{CA}$, the receiver altitude, $h$, and the signal grazing angle, $\gamma$ (see section 3.2 in Chapter I). Figure III.5 provides a graphical representation of the swaths covered by the various instruments onboard the NASA P3 in relation to the GPS radar footprint. Note that while the PSR and the LIDAR are both nadir looking instruments, the GPS radar allows for a greater range of signal incidence angles, which may result in poorly overlapping spatial footprints.

2.2 Flight scenarios

We now describe the conditions of each of the flights. The background images shown next are RADARSAT SAR backscatter maps, with a superimposed layer provided by the National Ice Center (NIC, [Dedrick et al., 2001], see Appendix L) encoding the sea ice conditions during the third week of March in 2003. Flight tracks are overlaid in yellow.

A – Barrow

This scenario (Figure III.6) spans a range of shorefast and drifting first year ice conditions, from smooth to heavily deformed [Sturm et al., 2006]. The sea ice concentration is nearly 100% at this time of the year. Inland, the most prominent features are the presence of (bright and dark) oblong frozen lakes and a vast expanse of (darkest) very smooth shorefast ice behind the thin land barrier that encloses the Elson Lagoon. NE of Point Barrow (in the Beaufort Sea) an area of large shear and pressure that reflects the
general easterly motion of the ice pack gives rise to bands of heavily rubbed ice and large ridges several meters high. NW of Point Barrow (in the Chukchi Sea), the drifting pack ice area shows variable patterns, with older first year floes floating on a rough matrix of first year ice [Maslanik et al., 2006]. Near Pt. Barrow, sea ice is typically about 1.2 to 1.5 meters thick in spring, with a snow cover typically ranging from 10 to 30 cm. While the shorefast ice in Elson Lagoon remains essentially stable and unaffected by significant ice movement, the ice along the Chukchi coast may form and break away multiple times, resulting in an ice that is less uniform in thickness and snow depth.

Figure III.6 – “Barrow” NIC analysis on SAR background

<table>
<thead>
<tr>
<th>Sector Label</th>
<th>Dominant Ice Type</th>
<th>Also Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>First Year</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Thin First Year</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Land</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Fast First Year</td>
<td>-</td>
</tr>
</tbody>
</table>
This is our highest latitude flight. During the time of the AMSR-E validation campaign, the US Navy operated an ice camp about 175 km northeast of Barrow (on a multiyear floe, at roughly 73° N, 147.5° E) in the main pack of the Beaufort Sea. The NIC ice analysis identifies four main sea ice sectors labeled 1 to 4 in Figure III.7. In all sectors, the ice concentration is nearly 100%, with the presence of a few (bright) multiyear ice floes interspersed with mostly first year ice [Sturm et al., 2006] in the northern sectors and (darker) thinner ice classes at the bottom of the image.

![Figure III.7 – “Icecamp” NIC analysis on SAR background](image)

### Table III.5 - “Icecamp” NIC classes

<table>
<thead>
<tr>
<th>Sector Label</th>
<th>Dominant Ice Type</th>
<th>Also Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Multiyear</td>
<td>First Year, Young</td>
</tr>
<tr>
<td>2</td>
<td>Multiyear</td>
<td>First Year</td>
</tr>
<tr>
<td>3</td>
<td>First Year</td>
<td>Multiyear</td>
</tr>
<tr>
<td>4</td>
<td>First Year</td>
<td>Thin First Year</td>
</tr>
</tbody>
</table>
This flight takes place south of the island of Saint Matthew, at about 60°N, where the sea ice cover meets the open ocean as of Spring 2003. Ice types range from thin first year in the northern sectors to open water at the bottom of the image.

The ice concentration is nearly 100% in the upper right sectors and decreases southwards in a gradual fashion. Observe the wake that the southward ice drift creates behind the island.

Figure III.8 – “Bering Sea” NIC analysis on SAR background

Table III.6 - “Bering Sea” NIC classes

<table>
<thead>
<tr>
<th>Sector Label</th>
<th>Dominant Ice Type</th>
<th>Also Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thin First Year</td>
<td>Young</td>
</tr>
<tr>
<td>2</td>
<td>Young</td>
<td>New Ice</td>
</tr>
<tr>
<td>3</td>
<td>Thin First Year</td>
<td>Young, Open Water</td>
</tr>
<tr>
<td>4</td>
<td>Belts of Thin FY</td>
<td>Young, Open Water</td>
</tr>
<tr>
<td>5</td>
<td>Open Water</td>
<td></td>
</tr>
</tbody>
</table>
This flight covers the mouth of the Norton Sound, south of the Bering Strait. Northerly winds in the area cause newly formed sea ice to be advected from the northern coastal polynya into the sound along the SE direction. Ice concentrations are nearly 100% in all sectors. According to the NIC analysis, the area under observation can be described as a matrix of young and thin ice types with first year ice patches.

<table>
<thead>
<tr>
<th>Sector Label</th>
<th>Dominant Ice Type</th>
<th>Also Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Land</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Young and Thin FY</td>
<td>New ice</td>
</tr>
<tr>
<td>3</td>
<td>Young</td>
<td>Thin First Year</td>
</tr>
<tr>
<td>4</td>
<td>First Year</td>
<td>Thin First Year</td>
</tr>
<tr>
<td>5</td>
<td>Fast First Year</td>
<td>Thin FY</td>
</tr>
<tr>
<td>6</td>
<td>Thin FY</td>
<td>Open water</td>
</tr>
</tbody>
</table>
**E – St Lawrence**

This flight takes place south of the island of St Lawrence, in the West Bering Sea. In the ice-covered early spring period, the area is influenced by a seasonal polynya, an area of open water that develops south of the island, as prevailing northerly winds force sea ice away from the land-mass [Kozo et al., 1990].

![Figure III.10 – “St Lawrence” NIC analysis on SAR background](image)

According to the NIC analysis, ice concentrations are about 90% in the southward side of the island, where (dark) young and new ice types ice classes are predominantly observed.

**Table III.8 - “St Lawrence” NIC classes**

<table>
<thead>
<tr>
<th>Sector Label</th>
<th>Dominant Ice Type</th>
<th>Also Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fast First Year</td>
<td>Thin First Year</td>
</tr>
<tr>
<td>2</td>
<td>Young</td>
<td>New Ice, Thin FY</td>
</tr>
<tr>
<td>3</td>
<td>Thin First Year</td>
<td>Young</td>
</tr>
<tr>
<td>4</td>
<td>First Year</td>
<td>Thin FY</td>
</tr>
<tr>
<td>5</td>
<td>Young</td>
<td>New Ice</td>
</tr>
<tr>
<td>6</td>
<td>Land</td>
<td></td>
</tr>
</tbody>
</table>
F – Point Hope

This area in the eastern Chukchi Sea spans a small coastal polynya that typically extends between Point Hope and Cape Lisburne. South of Point Hope, compression of the ice pack and extensive rafting is observed. According to the NIC analysis, the ice concentration is 90-100% in all sectors, with a mixture of new, young, thin first year and first year ice types.

Figure III.11 – “Point Hope” NIC analysis on SAR background

Table III.9 - “Point Hope” NIC classes

<table>
<thead>
<tr>
<th>Sector Label</th>
<th>Dominant Ice Type</th>
<th>Also Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thin First Year</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>First Year</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Land</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Thin First Year</td>
<td>Young</td>
</tr>
<tr>
<td>5</td>
<td>Young</td>
<td>New Ice</td>
</tr>
<tr>
<td>6</td>
<td>Fast First Year</td>
<td>-</td>
</tr>
</tbody>
</table>
This is our lowest latitude flight, about 58N in the Bering Sea. In conditions similar to those described for the St Matthews flight, the ice concentration is nearly 100% in upper left sectors, decreasing southwards in a gradual fashion. The main sea ice classes under observation are thin ice types (new ice, young and thin first year) and open water.

![Bering NIC analysis on SAR background](image)

**Figure III.12** - “Bering” NIC analysis on SAR background

**Table III.10** - “Bering” NIC classes

<table>
<thead>
<tr>
<th>Sector Label</th>
<th>Dominant Ice Type</th>
<th>Also Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thin First Year</td>
<td>Young</td>
</tr>
<tr>
<td>2</td>
<td>Thin FY</td>
<td>Young, Open Water</td>
</tr>
<tr>
<td>3</td>
<td>Young</td>
<td>Open Water</td>
</tr>
<tr>
<td>4</td>
<td>Open Water</td>
<td></td>
</tr>
</tbody>
</table>

### 2.3 Sea ice classification

Taken together, co-registered passive microwave, optical and SAR backscatter imagery make possible a reference sea ice classification co-located with the GPS bistatic data (see Table III.11 for an inspection of flight and satellite overpass dates). First, an unsupervised
clustering procedure (*k*-means on ENVI software, RSI Inc.) is used in the creation of sea ice classes that have similar passive microwave spectral/polarimetric signatures. The resulting classes are subject to an interpretation effort aided by SAR backscatter and MODIS optical images (see Table I.1) and merged into one of the following six thickness groups: open water (OW), new ice (NI), young ice (YI), thin first year ice (ThFYI), first year ice (FYI) and multiyear ice (MY). The characteristic thickness typically associated with each of these groups is shown in Table III.12 below. In the following subsections, we provide further details about the class clustering procedure and the criteria behind the interpretation effort, to end with a quantitative study of the separability of the thickness groups in terms of radiometric measurements alone.

Table III.11 – Time collocation between AMSRIce03 flights and satellite overpasses

<table>
<thead>
<tr>
<th>Flight</th>
<th>Date (mm/dd)</th>
<th>SAR overpass</th>
<th>MODIS overpass</th>
</tr>
</thead>
<tbody>
<tr>
<td>A – Point Barrow</td>
<td>03/13</td>
<td>– 2 days</td>
<td>+ 2 days</td>
</tr>
<tr>
<td>B – Icecamp</td>
<td>03/19</td>
<td>same day</td>
<td>same day</td>
</tr>
<tr>
<td>C – St Matthew</td>
<td>03/18</td>
<td>same day</td>
<td>– 2 days</td>
</tr>
<tr>
<td>D – Norton Sound</td>
<td>03/15</td>
<td>– 1 day</td>
<td>same day</td>
</tr>
<tr>
<td>E – St Lawrence</td>
<td>03/16</td>
<td>+ 1 day</td>
<td>same day</td>
</tr>
<tr>
<td>F – Point Hope</td>
<td>03/20</td>
<td>same day</td>
<td>same day</td>
</tr>
<tr>
<td>G – Bering Sea</td>
<td>03/22</td>
<td>– 2 days</td>
<td>same day</td>
</tr>
</tbody>
</table>

Table III.12 – Sea ice groups and associated thickness ranges

<table>
<thead>
<tr>
<th>Sea ice group</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>OW</td>
<td>0</td>
</tr>
<tr>
<td>NI</td>
<td>0-10 cm</td>
</tr>
<tr>
<td>YI</td>
<td>10-30 cm</td>
</tr>
<tr>
<td>ThFYI</td>
<td>30-70 cm</td>
</tr>
<tr>
<td>FYI</td>
<td>70-200 cm</td>
</tr>
<tr>
<td>MY</td>
<td>&gt; 200 cm</td>
</tr>
</tbody>
</table>

2.3.1 Emissivities

We assume that sea ice behaves as a blackbody and that measured brightness temperatures (for our purposes, in microwave as well as infrared bands) relate to the
physical temperature of the sea ice layer via Planck’s law (see Section 4.4 in Chapter II). At a given frequency and polarization, the brightness temperature measured by a radiometer is a sum of contributions arising from surface emission, downwelling atmospheric emission reflected by the surface, and upwelling atmospheric emission [Haggerty & Curry, 2001]. Assuming negligible atmospheric attenuation, the measured brightness temperature $T_b$ is given by the product of the surface emissivity times the surface physical temperature $T_{surf}$ as $T_b = \varepsilon_{surf} T_{surf}$. For the calculation of microwave emissivities, the surface physical (skin) temperature $T_{surf}$ is derived from the infrared radiance measured by the KT19 radiometer, corrected for the fact that sea ice is almost a perfect blackbody – we will use an average infrared emissivity for sea ice of 0.98 [Salisbury et al., 1994]. Due to weak absorption of microwaves by sea ice, thermal microwave emissions may emanate from a layer well under the air-snow interface and cause a difference between infrared and microwave skin temperatures when strong vertical gradients of temperature are present (e.g. for warmer sea ice under a colder thick snow cover). For typical winter conditions and a snow cover of about 20 cm deep, the snow-ice interface temperature can be 10 to 20 degrees higher than the air-snow temperature (~250 Kelvin, [Sturm et al., 2006]). Thus, the lack of direct measurements of the physical temperature of the emitting layer may contribute with a 4-8% error in the calculated emissivities.

2.3.2 Class clustering

Our approach begins with an unsupervised $k$-means classification (using ENVI software) on passive microwave channels. The $k$-means classification defines initial class means as evenly distributed in the data space and then iteratively clusters the pixels into the nearest class using a minimum distance technique [Richards & Xiuping, 1999]. Each iteration recalculates class means and reclassifies pixels relative to the new means. The
initialization of the iterative procedure requires specification of the number of clusters expected, which is chosen to be about 5 per flight. This number of expected clusters is chosen purposefully large, so that the typical variability of sea ice forms within thickness groups is well represented.

The maps that result from the unsupervised classification algorithm are shown next, along with a RGB composite image of the MODIS sensor for reference purposes (RGB for MODIS bands 4, 5 and 6 at 555, 1240 and 1640 microns respectively). The tables below summarize the average microwave and thermal infrared radiometric temperatures of each cluster, together with average SAR backscatter values, and polarization and spectral gradient ratios defined as in Chapter I, Section 4.2.2.

A – Barrow

![Figure III.13 – “Barrow” unsupervised classification](image)

<table>
<thead>
<tr>
<th>CLASS</th>
<th>TB11V</th>
<th>TB19V</th>
<th>TB37V</th>
<th>TB11H</th>
<th>TB19H</th>
<th>TB37H</th>
<th>IR</th>
<th>SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>249±1</td>
<td>246±1</td>
<td>235±3</td>
<td>233±3</td>
<td>230±3</td>
<td>222±4</td>
<td>251±2</td>
<td>-15±3</td>
</tr>
<tr>
<td>A2</td>
<td>250±1</td>
<td>248±1</td>
<td>240±2</td>
<td>230±2</td>
<td>229±2</td>
<td>224±3</td>
<td>250±2</td>
<td>-20±3</td>
</tr>
<tr>
<td>A3</td>
<td>250±1</td>
<td>249±1</td>
<td>246±2</td>
<td>231±2</td>
<td>230±2</td>
<td>230±2</td>
<td>250±2</td>
<td>-21±3</td>
</tr>
<tr>
<td>A4</td>
<td>249±1</td>
<td>248±1</td>
<td>246±2</td>
<td>235±2</td>
<td>234±2</td>
<td>237±3</td>
<td>252±2</td>
<td>-15±2</td>
</tr>
</tbody>
</table>
Table III.13b – Class statistics: polarization and gradient ratios

<table>
<thead>
<tr>
<th>CLASS</th>
<th>PR_{11}</th>
<th>PR_{19}</th>
<th>PR_{37}</th>
<th>GR_{11-19H}</th>
<th>GR_{11-19V}</th>
<th>GR_{19-37H}</th>
<th>GR_{19-37V}</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.033</td>
<td>0.034</td>
<td>0.028</td>
<td>-0.006</td>
<td>-0.006</td>
<td>-0.018</td>
<td>-0.023</td>
</tr>
<tr>
<td>A2</td>
<td>0.042</td>
<td>0.040</td>
<td>0.034</td>
<td>-0.002</td>
<td>-0.004</td>
<td>-0.011</td>
<td>-0.016</td>
</tr>
<tr>
<td>A3</td>
<td>0.040</td>
<td>0.040</td>
<td>0.034</td>
<td>0.0</td>
<td>-0.002</td>
<td>0.0</td>
<td>-0.006</td>
</tr>
<tr>
<td>A4</td>
<td>0.029</td>
<td>0.029</td>
<td>0.019</td>
<td>-0.002</td>
<td>-0.002</td>
<td>0.004</td>
<td>-0.006</td>
</tr>
</tbody>
</table>

Table III.13c – Class statistics: emissivities

<table>
<thead>
<tr>
<th>CLASS</th>
<th>ε(11V)</th>
<th>ε(19V)</th>
<th>ε(37V)</th>
<th>ε(11H)</th>
<th>ε(19H)</th>
<th>ε(37H)</th>
<th>sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.97</td>
<td>0.96</td>
<td>0.92</td>
<td>0.91</td>
<td>0.90</td>
<td>0.87</td>
<td>±0.01</td>
</tr>
<tr>
<td>A2</td>
<td>0.98</td>
<td>0.97</td>
<td>0.94</td>
<td>0.90</td>
<td>0.90</td>
<td>0.88</td>
<td>±0.01</td>
</tr>
<tr>
<td>A3</td>
<td>0.98</td>
<td>0.97</td>
<td>0.96</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>±0.01</td>
</tr>
<tr>
<td>A4</td>
<td>0.97</td>
<td>0.96</td>
<td>0.96</td>
<td>0.91</td>
<td>0.91</td>
<td>0.92</td>
<td>±0.01</td>
</tr>
</tbody>
</table>

Figure III.14 – “Icecamp” unsupervised classification

Table III.14a – Class statistics: brightness temperatures (K) and SAR backscatter (dB)

<table>
<thead>
<tr>
<th>CLASS</th>
<th>TB11V</th>
<th>TB19V</th>
<th>TB37V</th>
<th>TB11H</th>
<th>TB19H</th>
<th>TB37H</th>
<th>IR</th>
<th>SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>241±2</td>
<td>234±3</td>
<td>222±3</td>
<td>218±2</td>
<td>211±4</td>
<td>206±3</td>
<td>245±2</td>
<td>-13±3</td>
</tr>
<tr>
<td>B2</td>
<td>246±1</td>
<td>239±2</td>
<td>230±3</td>
<td>223±2</td>
<td>220±3</td>
<td>216±3</td>
<td>247±2</td>
<td>-16±2</td>
</tr>
<tr>
<td>B3</td>
<td>246±1</td>
<td>244±2</td>
<td>237±2</td>
<td>227±2</td>
<td>227±3</td>
<td>225±3</td>
<td>247±2</td>
<td>-18±4</td>
</tr>
<tr>
<td>B4</td>
<td>246±1</td>
<td>245±1</td>
<td>243±2</td>
<td>224±3</td>
<td>228±2</td>
<td>232±2</td>
<td>247±2</td>
<td>-21±3</td>
</tr>
<tr>
<td>B5</td>
<td>247±1</td>
<td>247±1</td>
<td>243±2</td>
<td>229±2</td>
<td>232±2</td>
<td>232±3</td>
<td>247±2</td>
<td>-20±3</td>
</tr>
</tbody>
</table>
Table III.14b – Class statistics: polarization and gradient ratios

<table>
<thead>
<tr>
<th>CLASS</th>
<th>PR11</th>
<th>PR19</th>
<th>PR37</th>
<th>GR11,19H</th>
<th>GR11,19V</th>
<th>GR19,37H</th>
<th>GR19,37V</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.050</td>
<td>0.051</td>
<td>0.037</td>
<td>-0.016</td>
<td>-0.015</td>
<td>-0.012</td>
<td>-0.026</td>
</tr>
<tr>
<td>B2</td>
<td>0.049</td>
<td>0.041</td>
<td>0.031</td>
<td>-0.007</td>
<td>-0.014</td>
<td>-0.009</td>
<td>-0.019</td>
</tr>
<tr>
<td>B3</td>
<td>0.040</td>
<td>0.036</td>
<td>0.026</td>
<td>0.000</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.014</td>
</tr>
<tr>
<td>B4</td>
<td>0.047</td>
<td>0.036</td>
<td>0.023</td>
<td>0.009</td>
<td>-0.002</td>
<td>0.009</td>
<td>-0.004</td>
</tr>
<tr>
<td>B5</td>
<td>0.038</td>
<td>0.031</td>
<td>0.023</td>
<td>0.006</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.008</td>
</tr>
</tbody>
</table>

Table III.14c – Class statistics: emissivities

<table>
<thead>
<tr>
<th>CLASS</th>
<th>ε(11V)</th>
<th>ε(19V)</th>
<th>ε(37V)</th>
<th>ε(11H)</th>
<th>ε(19H)</th>
<th>ε(37H)</th>
<th>sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0.96</td>
<td>0.93</td>
<td>0.88</td>
<td>0.87</td>
<td>0.84</td>
<td>0.82</td>
<td>±0.01</td>
</tr>
<tr>
<td>B2</td>
<td>0.97</td>
<td>0.95</td>
<td>0.91</td>
<td>0.88</td>
<td>0.87</td>
<td>0.86</td>
<td>±0.01</td>
</tr>
<tr>
<td>B3</td>
<td>0.98</td>
<td>0.97</td>
<td>0.94</td>
<td>0.90</td>
<td>0.90</td>
<td>0.89</td>
<td>±0.01</td>
</tr>
<tr>
<td>B4</td>
<td>0.97</td>
<td>0.97</td>
<td>0.96</td>
<td>0.89</td>
<td>0.90</td>
<td>0.92</td>
<td>±0.01</td>
</tr>
<tr>
<td>B5</td>
<td>0.98</td>
<td>0.98</td>
<td>0.96</td>
<td>0.91</td>
<td>0.92</td>
<td>0.92</td>
<td>±0.01</td>
</tr>
</tbody>
</table>

C – St Matthew

Figure III.15 – “St Matthew” unsupervised classification

Table III.15a – Class statistics: brightness temperatures (K) and SAR backscatter (dB)

<table>
<thead>
<tr>
<th>CLASS</th>
<th>TB11V</th>
<th>TB19V</th>
<th>TB37V</th>
<th>TB11H</th>
<th>TB19H</th>
<th>TB37H</th>
<th>IR</th>
<th>SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>182±8</td>
<td>193±7</td>
<td>212±5</td>
<td>115±10</td>
<td>126±9</td>
<td>151±7</td>
<td>270±2</td>
<td>-20±3</td>
</tr>
<tr>
<td>C2</td>
<td>224±12</td>
<td>227±9</td>
<td>237±8</td>
<td>159±10</td>
<td>164±9</td>
<td>186±10</td>
<td>266±2</td>
<td>-24±2</td>
</tr>
<tr>
<td>C3</td>
<td>241±7</td>
<td>242±5</td>
<td>247±5</td>
<td>179±9</td>
<td>182±8</td>
<td>200±9</td>
<td>264±2</td>
<td>-24±2</td>
</tr>
<tr>
<td>C4</td>
<td>245±5</td>
<td>246±5</td>
<td>246±6</td>
<td>206±9</td>
<td>210±7</td>
<td>218±7</td>
<td>263±2</td>
<td>-13±2</td>
</tr>
<tr>
<td>C5</td>
<td>253±3</td>
<td>252±2</td>
<td>246±4</td>
<td>227±6</td>
<td>228±5</td>
<td>229±5</td>
<td>260±2</td>
<td>-21±3</td>
</tr>
</tbody>
</table>
Table III.15b – Class statistics: polarization and gradient ratios

<table>
<thead>
<tr>
<th>CLASS</th>
<th>PR_{11}</th>
<th>PR_{19}</th>
<th>PR_{37}</th>
<th>GR_{11,19H}</th>
<th>GR_{11,19V}</th>
<th>GR_{19-37H}</th>
<th>GR_{19-37V}</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.226</td>
<td>0.210</td>
<td>0.168</td>
<td>0.046</td>
<td>0.029</td>
<td>0.090</td>
<td>0.047</td>
</tr>
<tr>
<td>C2</td>
<td>0.170</td>
<td>0.161</td>
<td>0.121</td>
<td>0.015</td>
<td>0.007</td>
<td>0.063</td>
<td>0.022</td>
</tr>
<tr>
<td>C3</td>
<td>0.148</td>
<td>0.142</td>
<td>0.105</td>
<td>0.008</td>
<td>0.002</td>
<td>0.047</td>
<td>0.010</td>
</tr>
<tr>
<td>C4</td>
<td>0.086</td>
<td>0.079</td>
<td>0.060</td>
<td>0.010</td>
<td>0.002</td>
<td>0.019</td>
<td>0.0</td>
</tr>
<tr>
<td>C5</td>
<td>0.054</td>
<td>0.050</td>
<td>0.036</td>
<td>0.002</td>
<td>-0.002</td>
<td>0.002</td>
<td>-0.012</td>
</tr>
</tbody>
</table>

Table III.15c – Class statistics: emissivities

<table>
<thead>
<tr>
<th>CLASS</th>
<th>$\epsilon_{(11V)}$</th>
<th>$\epsilon_{(19V)}$</th>
<th>$\epsilon_{(37V)}$</th>
<th>$\epsilon_{(11H)}$</th>
<th>$\epsilon_{(19H)}$</th>
<th>$\epsilon_{(37H)}$</th>
<th>sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.66</td>
<td>0.70</td>
<td>0.77</td>
<td>0.41</td>
<td>0.46</td>
<td>0.69</td>
<td>±0.03</td>
</tr>
<tr>
<td>C2</td>
<td>0.83</td>
<td>0.84</td>
<td>0.87</td>
<td>0.58</td>
<td>0.60</td>
<td>0.69</td>
<td>±0.03</td>
</tr>
<tr>
<td>C3</td>
<td>0.90</td>
<td>0.92</td>
<td>0.92</td>
<td>0.66</td>
<td>0.68</td>
<td>0.74</td>
<td>±0.03</td>
</tr>
<tr>
<td>C4</td>
<td>0.91</td>
<td>0.92</td>
<td>0.92</td>
<td>0.76</td>
<td>0.78</td>
<td>0.81</td>
<td>±0.03</td>
</tr>
<tr>
<td>C5</td>
<td>0.95</td>
<td>0.95</td>
<td>0.93</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>±0.02</td>
</tr>
</tbody>
</table>

Figure III.16 – “Norton Sound” unsupervised classification

Table III.16a – Class statistics: brightness temperatures (K) and SAR backscatter (dB)

<table>
<thead>
<tr>
<th>CLASS</th>
<th>TB_{11V}</th>
<th>TB_{19V}</th>
<th>TB_{37V}</th>
<th>TB_{11H}</th>
<th>TB_{19H}</th>
<th>TB_{37H}</th>
<th>IR</th>
<th>SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>251±5</td>
<td>243±4</td>
<td>218±6</td>
<td>210±11</td>
<td>202±8</td>
<td>191±6</td>
<td>254±4</td>
<td>-19±3</td>
</tr>
<tr>
<td>D2</td>
<td>253±2</td>
<td>247±2</td>
<td>227±4</td>
<td>218±5</td>
<td>212±4</td>
<td>203±4</td>
<td>255±3</td>
<td>-17±2</td>
</tr>
<tr>
<td>D3</td>
<td>246±6</td>
<td>246±4</td>
<td>243±6</td>
<td>201±14</td>
<td>207±8</td>
<td>212±6</td>
<td>259±4</td>
<td>-25±1</td>
</tr>
<tr>
<td>D4</td>
<td>253±2</td>
<td>249±2</td>
<td>236±4</td>
<td>225±8</td>
<td>222±4</td>
<td>216±4</td>
<td>256±3</td>
<td>-16±2</td>
</tr>
</tbody>
</table>

D – Norton Sound
Table III.16b – Class statistics: polarization and gradient ratios

<table>
<thead>
<tr>
<th>CLASS</th>
<th>PR_{11}</th>
<th>PR_{19}</th>
<th>PR_{37}</th>
<th>GR_{11-19H}</th>
<th>GR_{11-19V}</th>
<th>GR_{19-37H}</th>
<th>GR_{19-37V}</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>0.091</td>
<td>0.095</td>
<td>0.066</td>
<td>-0.019</td>
<td>-0.016</td>
<td>-0.028</td>
<td>-0.054</td>
</tr>
<tr>
<td>D2</td>
<td>0.079</td>
<td>0.076</td>
<td>0.056</td>
<td>-0.014</td>
<td>-0.012</td>
<td>-0.022</td>
<td>-0.042</td>
</tr>
<tr>
<td>D3</td>
<td>0.101</td>
<td>0.086</td>
<td>0.068</td>
<td>0.015</td>
<td>0.0</td>
<td>0.012</td>
<td>-0.006</td>
</tr>
<tr>
<td>D4</td>
<td>0.061</td>
<td>0.057</td>
<td>0.044</td>
<td>-0.007</td>
<td>-0.008</td>
<td>-0.014</td>
<td>-0.027</td>
</tr>
</tbody>
</table>

Table III.16c – Class statistics: emissivities

<table>
<thead>
<tr>
<th>CLASS</th>
<th>(\varepsilon)_{11V}</th>
<th>(\varepsilon)_{19V}</th>
<th>(\varepsilon)_{37V}</th>
<th>(\varepsilon)_{11H}</th>
<th>(\varepsilon)_{19H}</th>
<th>(\varepsilon)_{37H}</th>
<th>sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>0.97</td>
<td>0.94</td>
<td>0.84</td>
<td>0.81</td>
<td>0.78</td>
<td>0.74</td>
<td>\pm0.03</td>
</tr>
<tr>
<td>D2</td>
<td>0.97</td>
<td>0.95</td>
<td>0.87</td>
<td>0.84</td>
<td>0.82</td>
<td>0.78</td>
<td>\pm0.03</td>
</tr>
<tr>
<td>D3</td>
<td>0.93</td>
<td>0.93</td>
<td>0.92</td>
<td>0.76</td>
<td>0.78</td>
<td>0.80</td>
<td>\pm0.03</td>
</tr>
<tr>
<td>D4</td>
<td>0.97</td>
<td>0.95</td>
<td>0.90</td>
<td>0.86</td>
<td>0.85</td>
<td>0.82</td>
<td>\pm0.03</td>
</tr>
</tbody>
</table>

**E – St Lawrence**

![Image of St Lawrence classification]

Figure III.17 – “St Lawrence” unsupervised classification

Table III.17a – Class statistics: brightness temperatures (K) and SAR backscatter (dB)

<table>
<thead>
<tr>
<th>CLASS</th>
<th>TB_{11V}</th>
<th>TB_{19V}</th>
<th>TB_{37V}</th>
<th>TB_{11H}</th>
<th>TB_{19H}</th>
<th>TB_{37H}</th>
<th>IR</th>
<th>SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>198±14</td>
<td>208±13</td>
<td>224±12</td>
<td>118±15</td>
<td>129±20</td>
<td>161±18</td>
<td>267±2</td>
<td>-23±1</td>
</tr>
<tr>
<td>E2</td>
<td>238±7</td>
<td>246±6</td>
<td>253±5</td>
<td>162±10</td>
<td>182±11</td>
<td>207±10</td>
<td>264±2</td>
<td>-25±1</td>
</tr>
<tr>
<td>E3</td>
<td>247±10</td>
<td>254±9</td>
<td>254±6</td>
<td>183±17</td>
<td>200±14</td>
<td>203±11</td>
<td>260±4</td>
<td>-25±1</td>
</tr>
</tbody>
</table>
Table III.17b – Class statistics: polarization and gradient ratios

<table>
<thead>
<tr>
<th>CLASS</th>
<th>PR_{11}</th>
<th>PR_{19}</th>
<th>PR_{37}</th>
<th>GR_{11-19H}</th>
<th>GR_{11-19V}</th>
<th>GR_{19-37H}</th>
<th>GR_{19-37V}</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>0.255</td>
<td>0.240</td>
<td>0.167</td>
<td>0.044</td>
<td>0.025</td>
<td>0.110</td>
<td>0.037</td>
</tr>
<tr>
<td>E2</td>
<td>0.191</td>
<td>0.150</td>
<td>0.100</td>
<td>0.058</td>
<td>0.016</td>
<td>0.065</td>
<td>0.014</td>
</tr>
<tr>
<td>E3</td>
<td>0.149</td>
<td>0.119</td>
<td>0.112</td>
<td>0.043</td>
<td>0.014</td>
<td>0.007</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table III.17c – Class statistics: emissivities

<table>
<thead>
<tr>
<th>CLASS</th>
<th>(\varepsilon) (11V)</th>
<th>(\varepsilon) (19V)</th>
<th>(\varepsilon) (37V)</th>
<th>(\varepsilon) (11H)</th>
<th>(\varepsilon) (19H)</th>
<th>(\varepsilon) (37H)</th>
<th>sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>0.73</td>
<td>0.76</td>
<td>0.82</td>
<td>0.43</td>
<td>0.47</td>
<td>0.59</td>
<td>(\pm 0.05)</td>
</tr>
<tr>
<td>E2</td>
<td>0.88</td>
<td>0.91</td>
<td>0.94</td>
<td>0.60</td>
<td>0.68</td>
<td>0.77</td>
<td>(\pm 0.04)</td>
</tr>
<tr>
<td>E3</td>
<td>0.92</td>
<td>0.95</td>
<td>0.95</td>
<td>0.68</td>
<td>0.74</td>
<td>0.75</td>
<td>(\pm 0.04)</td>
</tr>
</tbody>
</table>

\(F \rightarrow \text{Point Hope}\)

![Image showing Point Hope unsupervised classification]

Figure III.18 – “Point Hope” unsupervised classification

Table III.18a – Class statistics: brightness temperatures (K) and SAR backscatter (dB)

<table>
<thead>
<tr>
<th>CLASS</th>
<th>TB_{11}</th>
<th>TB_{19}</th>
<th>TB_{37}</th>
<th>TB_{11H}</th>
<th>TB_{19H}</th>
<th>TB_{37H}</th>
<th>IR</th>
<th>SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>205\pm 20</td>
<td>211\pm 16</td>
<td>223\pm 12</td>
<td>137\pm 20</td>
<td>143\pm 18</td>
<td>164\pm 14</td>
<td>260\pm 5</td>
<td>N/A</td>
</tr>
<tr>
<td>F2</td>
<td>242\pm 5</td>
<td>243\pm 4</td>
<td>244\pm 3</td>
<td>192\pm 7</td>
<td>196\pm 7</td>
<td>208\pm 6</td>
<td>254\pm 3</td>
<td>-24\pm 2</td>
</tr>
<tr>
<td>F3</td>
<td>242\pm 4</td>
<td>241\pm 4</td>
<td>241\pm 5</td>
<td>211\pm 6</td>
<td>214\pm 5</td>
<td>221\pm 6</td>
<td>253\pm 3</td>
<td>-14\pm 2</td>
</tr>
<tr>
<td>F4</td>
<td>249\pm 2</td>
<td>248\pm 1</td>
<td>244\pm 5</td>
<td>230\pm 4</td>
<td>229\pm 3</td>
<td>228\pm 6</td>
<td>250\pm 2</td>
<td>-18\pm 3</td>
</tr>
<tr>
<td>F5</td>
<td>244\pm 2</td>
<td>244\pm 2</td>
<td>244\pm 2</td>
<td>227\pm 4</td>
<td>228\pm 4</td>
<td>233\pm 3</td>
<td>251\pm 2</td>
<td>-15\pm 3</td>
</tr>
</tbody>
</table>
Table III.18b – Class statistics: polarization and gradient ratios

<table>
<thead>
<tr>
<th>CLASS</th>
<th>PR\textsubscript{11}</th>
<th>PR\textsubscript{19}</th>
<th>PR\textsubscript{37}</th>
<th>GR\textsubscript{11,19H}</th>
<th>GR\textsubscript{11,19V}</th>
<th>GR\textsubscript{19,37H}</th>
<th>GR\textsubscript{19,37V}</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>0.202</td>
<td>0.197</td>
<td>0.153</td>
<td>0.023</td>
<td>0.014</td>
<td>0.068</td>
<td>0.028</td>
</tr>
<tr>
<td>F2</td>
<td>0.116</td>
<td>0.106</td>
<td>0.080</td>
<td>0.012</td>
<td>-0.002</td>
<td>0.030</td>
<td>0.002</td>
</tr>
<tr>
<td>F3</td>
<td>0.067</td>
<td>0.060</td>
<td>0.044</td>
<td>0.006</td>
<td>-0.002</td>
<td>0.016</td>
<td>0.0</td>
</tr>
<tr>
<td>F4</td>
<td>0.040</td>
<td>0.040</td>
<td>0.034</td>
<td>-0.002</td>
<td>-0.002</td>
<td>-0.002</td>
<td>-0.008</td>
</tr>
<tr>
<td>F5</td>
<td>0.036</td>
<td>0.034</td>
<td>0.023</td>
<td>0.002</td>
<td>0.0</td>
<td>0.011</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table III.18c – Class statistics: emissivities

<table>
<thead>
<tr>
<th>CLASS</th>
<th>e\textsubscript{(11V)}</th>
<th>e\textsubscript{(19V)}</th>
<th>e\textsubscript{(37V)}</th>
<th>e\textsubscript{(11H)}</th>
<th>e\textsubscript{(19H)}</th>
<th>e\textsubscript{(37H)}</th>
<th>sigma</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>0.77</td>
<td>0.80</td>
<td>0.84</td>
<td>0.52</td>
<td>0.54</td>
<td>0.62</td>
<td>±0.08</td>
</tr>
<tr>
<td>F2</td>
<td>0.93</td>
<td>0.94</td>
<td>0.94</td>
<td>0.74</td>
<td>0.76</td>
<td>0.80</td>
<td>±0.02</td>
</tr>
<tr>
<td>F3</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.82</td>
<td>0.83</td>
<td>0.86</td>
<td>±0.02</td>
</tr>
<tr>
<td>F4</td>
<td>0.98</td>
<td>0.97</td>
<td>0.95</td>
<td>0.90</td>
<td>0.90</td>
<td>0.89</td>
<td>±0.02</td>
</tr>
<tr>
<td>F5</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.88</td>
<td>0.89</td>
<td>0.91</td>
<td>±0.02</td>
</tr>
</tbody>
</table>

$G$ – Bering

Figure III.19 – “Bering” unsupervised classification

Table III.19a – Class statistics: brightness temperatures (K) and SAR backscatter (dB)

<table>
<thead>
<tr>
<th>CLASS</th>
<th>TB\textsubscript{11V}</th>
<th>TB\textsubscript{19V}</th>
<th>TB\textsubscript{37V}</th>
<th>TB\textsubscript{11H}</th>
<th>TB\textsubscript{19H}</th>
<th>TB\textsubscript{37H}</th>
<th>IR</th>
<th>SAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>177±4</td>
<td>188±4</td>
<td>210±3</td>
<td>108±5</td>
<td>119±6</td>
<td>148±4</td>
<td>270±1</td>
<td>-23±3</td>
</tr>
<tr>
<td>G2</td>
<td>204±7</td>
<td>213±7</td>
<td>228±7</td>
<td>142±10</td>
<td>156±10</td>
<td>178±12</td>
<td>268±2</td>
<td>-23</td>
</tr>
<tr>
<td>G3</td>
<td>228±9</td>
<td>234±7</td>
<td>245±6</td>
<td>173±10</td>
<td>187±10</td>
<td>210±10</td>
<td>266±2</td>
<td>-24</td>
</tr>
<tr>
<td>G4</td>
<td>243±6</td>
<td>247±4</td>
<td>251±2</td>
<td>208±13</td>
<td>218±10</td>
<td>232±7</td>
<td>264±2</td>
<td>-14±3</td>
</tr>
</tbody>
</table>
2.3.3 Class interpretation

We begin this section with an overview of the general criteria that will guide our interpretation of the microwave signatures of unsupervised class clusters. A detailed analysis for each flight is given next and the resulting conclusions are wrapped up in a final summary.

General interpretation criteria

The qualitative interpretation of radiometric signatures relies on extensive field observations previously reported in the literature. Here we review the main radiometric traits of the most common sea ice forms in the Arctic based on these studies.

In general, open water is less emissive than sea ice in the microwave and it appears radiometrically colder. The OW signature is also characterized by strong polarization at angles near the Brewster angle for total V-pol transmission, more so as the dielectric jump at the boundary increases (see Figure III.20). The steady decrease in dielectric permittivity of seawater with frequency due to the water dipole effect [Debye, 1929] is
responsible for enhanced emissivities at larger frequencies and strong positive spectral gradients.

In contrast to seawater, the permittivity of sea ice is practically flat across the microwave spectrum [Hallikainen & Winebrenner, 1992]. The progressive growth of the sea ice layer, which acts to gradually shield the underlying seawater signature, is typically accompanied by a steady increase in brightness temperatures and decrease in polarization and spectral gradient ratios ([Grenfell et al., 1988], [Eppler et al., 1992], [Cavalieri, 1994]). The emissivity of new ice is characterized by relatively low emissivities, strong polarization and positive spectral gradients, particularly at H-pol, along with generally low backscatter due to surface damping. The emissivity of young ice peaks after the underlying seawater is covered with ice up to some thickness\(^2\), showing moderate polarization ratios and moderately positive H-pol spectral gradients but already null V-pol spectral gradients, along with generally high SAR backscatter due to high saline content. Once the sea ice slab has achieved its peak emissivity, further variability among the thin FYI, FYI and MYI ice forms results from differences in scattering depth and

\(^2\) About 30 cm thick, which roughly corresponds to the sea ice optical depth at 11 GHz.
characteristics of the snowpack and snow-ice interface. Rough surface scattering tends to increase the emissivity of very reflective surfaces [Eom, 1981] and slightly decrease the polarization ratios ([Hallikainen & Winebrenner, 1992], [Barber et al., 1998]). A layer of snow, when optically thin, will tend to increase the emissivity of the ice slab by acting as a layer of intermediate permittivity (impedance matching effect). When optically thick, the snow cover will attenuate the underlying layer transmittance, leading to lower radiometric temperatures in the presence of vertical temperature gradients (scatter darkening effect). Another factor affecting the emissivity of thin ice types is the formation of a liquid brine layer on top of the ice surface. The absorption of radiation in a highly lossy brine layer (which becomes optically thicker at higher frequencies) has the potential to significantly lower the radiometric temperature, increase the polarization and decrease the spectral gradient, but it is a factor that tends to be neglected. A similar effect is produced by volume scattering in the upper porous layer of the less emissive multiyear ice, which also leads to marked negative spectral gradients [Grenfell & Perovich, 1994].

Figure III.13 – Barrow flight

Class A4 is the warmest ice form in the Barrow scene, with a rather null spectral gradient, low polarization and high SAR backscatter, which are all characteristics of thin FYI/FYI ice with a certain degree of deformation –i.e. a (thin) deformed FYI type. Classes A1, A2 and A3 separate radiometrically at 37 GHz: Class A3 is warm, with a relatively large polarization ratio, low SAR backscatter and null spectral gradient ratio, which are characteristics of smooth FYI ice. Both Class A2 and Class A3 are shorefast types with similar SAR backscatter and brightness temperatures at 11 GHz, but Class A2 shows a more negative spectral gradient, which is usually associated with the presence of a snow cover. Class A1 is the coldest ice form in the scene, with a strong negative spectral gradient. Considering its high SAR backscatter, relatively low polarization and location along the coastline, we conclude that this is a shorefast deformed FYI type.
The clustering algorithm for the Icecamp flight identifies Classes B1 and B2 as having strong negative gradient ratios, which is characteristic of multiyear ice. Observe that this effect is detected at lower frequencies than that produced by snow or surface scattering alone (cf. Class A1 and Class A2 in Barrow). Note also that the negative spectral gradient of multiyear ice here is associated with polarization ratios slightly larger than those of first year ice (an increase that might be related to enhanced V-pol emissivities after vertical brine anisotropies are washed out). Within the multiyear ice types, Class B1 is radiometrically colder than Class B2, with higher SAR backscatter, larger polarization and stronger negative spectral gradient, probably due to a thicker porous layer (i.e. older multiyear). Classes B3 through B5 are relatively warmer, have large polarization ratios, low backscatter and null to positive spectral gradients at 11-19 GHz, all characteristics of first year ice. Class B3 has a negative spectral gradient at 19-37 GHz and higher SAR backscatter, which we take as an indication of roughness and attribute this class to deformed FYI. Classes B4 and B5 have both low backscatter, typical of smooth surfaces, but Class B4 is radiometrically colder, with a larger polarization and a more positive spectral gradient than Class B5 at low frequencies, which we take as an indication of thinness (i.e. the larger penetration at the lower frequencies is able to reach the cold and highly polarized water below). Therefore we label Class B4 as thin FYI and Class B5 as smooth FYI.

Open water is typically the radiometrically coldest and most polarized surface observed in the Arctic, which here corresponds to Class C1. It is also characterized by a large positive spectral gradient. Classes C2 through C4 are characteristic of developing thin ice, as seen in a progressive increase of H-pol emissivities along with a steady decrease of
spectral gradient and polarization ratios. Classes C2 and C3 have both very low SAR backscatter, which we attribute to the typical dampening of surface waves in *new ice*. Class C4 has a much higher SAR backscatter, which we relate to a permittivity increase due to brine ejection onto the surface of *young ice*. Class C5 is similar to Class B4 in Icecamp, only with slightly lower emissivity, higher polarization and more negative spectral gradient. Thus we attribute this class the label of “lightly deformed” *thin FYI*.

*Figure III.16 – Norton Sound flight*

Relatively high emissivities at 11 GHz and high polarization ratios in this scene are indicative of thin ice types, although anomalously large negative gradients for all classes (except for Class D3) are reminiscent of multiyear ice. False indications of multiyear ice have been previously reported in new/young ice areas [Cavalieri, 1994], thin FYI areas in the Baltic Sea [Hewison, 1999] and areas with a substantial snow cover in the Southern Oceans [Markus & Cavalieri, 2000]. The lower emissivity at higher frequencies is characteristic of volume scattering in air/brine pockets within the older ice pack, but may also be associated with volume scattering by a thick snow cover (which would tend to decrease the PR, [Barber et al., 1998]) and/or absorption by a thin saline brine layer (which would tend to increase the polarization ratio by increasing the H-pol reflectivity). Thus we suggest that the negative spectral gradients of these thin ice types are a consequence of brine ejection onto the surface following a documented melt/refreeze episode in the area. The highest polarization and lowest radiometric temperature in the scene correspond to Class D3, which also shows a positive spectral gradient and low SAR backscatter. These are all characteristic of *new/young ice*. Classes D1, D2 and D4 come next in a scale of decreasing polarization, with increasing SAR backscatter and increasing radiometric temperature, which we attribute to thin FYI types with increasing degrees of surface roughness.
Class E1 is radiometrically cold, strongly polarized and with a positive spectral signature typical of open water. Classes E2 and E3 are slightly warmer, still strongly polarized and with positive spectral gradients (especially for H-pol components), which we take as indicative of new/young ice.

With the highest polarization, highest positive spectral gradient and lowest brightness temperature, Class F1 can be labeled as open water. The remaining classes have all similar V-pol temperatures but can be separated into distinct thin ice types in terms of their increasing H-pol temperatures and decreasing H-pol spectral gradients (in agreement with the thin ice types observed in St Matthew). Classes F2 and F3 have similarly cool H-pol radiometric temperatures (young ice) although Class F2 shows a much lower backscatter than Class F3. Classes F4 and F5 are radiometrically warmer, with small polarization ratios and slight negative and positive spectral gradients respectively, which we attribute to shorefast FY and thin FYI. The SAR backscatter is moderate for Class F4 (shorefast FYI) along the coast, while considerably higher for Class F5 (thin FYI).

Class G1 has a signature typical of open water, namely low radiometric temperature, high polarization and strong positive spectral gradient. Class G4 has a signature more typical of young ice (i.e. moderate polarization and positive spectral gradient) along with high SAR backscatter and a streamer-like appearance. Classes G2 and G3 signatures lie somewhere between that of open water (Class G1) and young ice (Class G4), and thus we label them as open water the former and new ice the latter.
Summary

A summary of the identified cluster characteristics and interpretation labels is given in the table and figures below. The radiometric signatures at 11, 19 and 37 GHz of the different sea ice groups are displayed in the following figures, reflecting the general criteria that have guided our interpretation. The separability of the radiometric classes is further investigated using a polarization/spectral gradient ($PR_{19}$-$GR_{19:37}$) scatter plot, which is the basis of the current algorithms for the discrimination of FY and MY ice types (NASA Team Algorithm, [Markus & Cavalieri, 2000]).

<table>
<thead>
<tr>
<th>FLIGHT</th>
<th>CLASS</th>
<th>Class label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrow</td>
<td>A1</td>
<td>Shorefast deformed FYI</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>Shorefast FYI</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>Shorefast FYI</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>Deformed Thin FYI</td>
</tr>
<tr>
<td>Icecamp</td>
<td>B1</td>
<td>MYI</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>MYI</td>
</tr>
<tr>
<td></td>
<td>B3</td>
<td>Deformed FYI</td>
</tr>
<tr>
<td></td>
<td>B4</td>
<td>Thin FYI</td>
</tr>
<tr>
<td></td>
<td>B5</td>
<td>Smooth FYI</td>
</tr>
<tr>
<td>St Matthew</td>
<td>C1</td>
<td>Open water</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>New ice</td>
</tr>
<tr>
<td></td>
<td>C3</td>
<td>New ice</td>
</tr>
<tr>
<td></td>
<td>C4</td>
<td>Young ice</td>
</tr>
<tr>
<td></td>
<td>C5</td>
<td>Thin FYI</td>
</tr>
<tr>
<td>Norton Sound</td>
<td>D1</td>
<td>Thin FYI</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>Thin FYI</td>
</tr>
<tr>
<td></td>
<td>D3</td>
<td>Young ice</td>
</tr>
<tr>
<td></td>
<td>D4</td>
<td>Thin FYI</td>
</tr>
<tr>
<td>St Lawrence</td>
<td>E1</td>
<td>Open water</td>
</tr>
<tr>
<td></td>
<td>E2</td>
<td>New ice</td>
</tr>
<tr>
<td></td>
<td>E3</td>
<td>New ice</td>
</tr>
<tr>
<td>Point Hope</td>
<td>F1</td>
<td>Open water</td>
</tr>
<tr>
<td></td>
<td>F2</td>
<td>Young ice</td>
</tr>
<tr>
<td></td>
<td>F3</td>
<td>Young ice</td>
</tr>
<tr>
<td></td>
<td>F4</td>
<td>Shorefast FYI</td>
</tr>
<tr>
<td></td>
<td>F5</td>
<td>Thin FYI</td>
</tr>
<tr>
<td>Bering</td>
<td>G1</td>
<td>Open water</td>
</tr>
<tr>
<td></td>
<td>G2</td>
<td>Open water</td>
</tr>
<tr>
<td></td>
<td>G3</td>
<td>New ice</td>
</tr>
<tr>
<td></td>
<td>G4</td>
<td>Young ice</td>
</tr>
</tbody>
</table>
Figure III.21a – Polarization/spectral gradient scatter plot

Figure III.21b – Emissivities of different radiometric classes
We observe that the evolution of open water into consolidated ice forms (OW, NI and YI) is clearly presented as a progressive decrease in polarization and spectral gradient ratios up to the thin FYI group. Further on, the radiometric separation among consolidated thin FY, FY and MY groups is compromised by a large degree of overlap among classes, indicating that their unambiguous identification using passive microwave data may be problematic.

For a more quantitative approach to radiometric separability, we define a metric based on spectrally averaged differences in emissivity between sea ice clusters \((a, b)\) as:

\[
\Delta \varepsilon_{a,b} = \sqrt{\sum_{\text{channels}} (\varepsilon_{a,i} - \varepsilon_{b,i})^2 / N_{\text{channels}}}
\]

Average distances between labeled clusters are shown in Table III.21 below. The typical size of the observed class clusters is \(\Delta \varepsilon_{a,a} \sim 0.05\), meaning that if cluster distances are larger/smaller than the typical class cluster size, then clusters become distinct/similar. Based on spectrally averaged differences in emissivity, we observe that intra-group similarities are reasonable (e.g., that thin first year ice observed in St Matthews is similar to thin first year ice observed in Point Hope). However, a significant degree of unexpected similarity is observed among the thin FYI, FYI and MYI groups on the lower right corner of the table. Although the separation between these groups has been based on a multisensor interpretation approach, their separability seems not sufficiently well reflected in terms of passive microwave measurements alone.
Table III.21 - Class separability in terms of spectrally averaged emissivity differences (x100)

![Table and Diagram]
In this section, surface height profiles collected by the ATM LIDAR profiler [Krabill et al., 2002] are used to characterize the large scale surface roughness of sea ice over a variety of Arctic scenarios [Belmonte et al., 2006]. After mean height removal, the LIDAR sea ice profiles are Fourier transformed to obtain the power spectral density of roughness within the profiler sampling bandpass. The spectral behavior of sea ice roughness is found to follow a lorentzian curve, allowing a simple parameterization of the sea ice “large-scale” roughness in terms of exponential root mean square height and correlation length, for length scales that range from 1 to 100 meters.

\textit{Power spectral densities}

After mean height removal, every conical profile becomes an isotropic realization of sea ice roughness. To characterize such a zero-mean process, second order statistics such as the autocorrelation function (ACF) are typically used. For a \textit{stationary} process, the Fourier transform of the autocorrelation function of heights gives the roughness power spectral density (PSD). For a one-dimensional surface profile $z(x)$, where $x$ represents the position of the LIDAR footprint in a conical profile of length $L$ and $N$ samples, the roughness spectral density $PSD(f)$ is estimated in terms of finite and discrete Fourier transforms $Z(f)$ of the profiles $z(x)$ as:

$$PSD(f) = |Z(f)|^2$$

(2.4.1)

Where $f = k \Delta f$, $\Delta f = 1/L$, $k = 1, \ldots, N$ and the Fourier transform pairs are defined as:

$$Z(k) = \frac{1}{N} \sum_{n=1}^{N} z(n) \exp(-j2\pi kn/N) \quad k = 1, \ldots, N$$

(2.4.2)

$$z(n) = \sum_{k=1}^{N} Z(k) \exp(j2\pi kn/N) \quad n = 1, \ldots, N$$
Where \( x = n \Delta x \) and \( \Delta x = L/N \). PSD estimates are averaged over periods of 1 second to increase their statistical confidence. According to the Nyquist theorem, the maximum and minimum spatial frequencies of roughness contained in the PSDs are dependent on the profile sample spacing \( \Delta x \) and length \( L \), and given by \( f_{\text{max}} = 1/(2\Delta x) \) and \( f_{\text{min}} = 1/L \). The profiler sampling bandpass \([f_{\text{min}}, f_{\text{max}}]\) determines the length-scales of roughness that are observable to the ATM sensor. The mean square height \( \sigma^2 \) and mean square slope \( \text{mss} \) of sea ice roughness can be calculated either directly from surface profiles \( z(x) \) [Bennett & Mattsson, 1989] or via the spectral densities \( PSD(f) \) [Bendat & Piersol, 1971]. In the spatial domain:

\[
\sigma^2 = \frac{1}{N} \sum_{n=1}^{N} z(n)^2 \quad \text{mss} = \frac{1}{N} \sum_{n=1}^{N} \left[ (z(n+1) - z(n))/\Delta x \right]^2 \quad (2.4.3)
\]

And in the spectral domain:

\[
\sigma^2 = 2 \sum_{k=1,N/2} \text{PSD}(k) \quad \text{mss} = 2 \sum_{k=1,N/2} (2\pi k/L)^2 \text{PSD}(k) \quad (2.4.4)
\]

These expressions become, in passing to the continuous limit with \( N \to \infty, \Delta x \to 0, N\Delta x \to L \):

\[
\sigma^2 = 2L \int_{f_{\text{min}}}^{f_{\text{max}}} PSD(f) df \quad \text{mss} = 2L \int_{f_{\text{min}}}^{f_{\text{max}}} (2\pi f)^2 PSD(f) df \quad (2.4.5)
\]

The roughness mean square height is represented by the area enclosed under the PSD curve, whereas the \( \text{mss} \) is given by that same area weighted by a factor of \((2\pi f)^2\). The spectral domain representation in illustrates the dependence of roughness statistics on the width and location of the sensor sampling bandpass \([f_{\text{min}}, f_{\text{max}}]\), which in the current ATM configuration is dependent on aircraft altitude (see Table III.22).

<table>
<thead>
<tr>
<th>Bandpass</th>
<th>( f_{\text{min}}/f_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low altitude (h=150m)</td>
<td>( \Delta x = 1 \text{ m} : L = 250 \text{ m} )</td>
</tr>
<tr>
<td>High altitude (h=1200m)</td>
<td>( \Delta x = 8 \text{ m} : L = 2000 \text{ m} )</td>
</tr>
</tbody>
</table>
In order to construct roughness descriptors that remain independent of aircraft altitude, the following scheme is proposed: observed high and low altitude roughness PSDs are fit to model functions which are then extrapolated to a common “large scale” sampling bandpass, with a fixed virtual sampling distance and a fixed virtual profile length, to calculate altitude-independent RMS height and $mss$ descriptors using (2.4.5).

The most salient feature in measured PSDs is that they decay in a lorentzian fashion, or equivalently, that their autocorrelation resembles an exponential function within the range of scales available to the ATM profiler (see Figure III.22). Fitting observed PSDs to lorentzian curves allows a simple characterization of sea ice roughness via exponential ACF parameters, $\sigma_b$ and $l_b$, which stand for “band-limited” root mean square height and correlation length respectively. The roughness ACF and PSD functions are Fourier pairs defined by:

\[ ACF(x) = \sigma_b^2 \exp(-|x|/l_b) \]  
\[ (2.4.6) \]

\[ PSD(f) = \frac{1}{N} \sum_{i=1}^{N} ACF(x) \exp\left(\frac{-2\pi n f}{L}\right) \rightarrow \frac{1}{L} \int_{0}^{L} ACF(x) \cos(2\pi f x) dx = \frac{2\sigma_l^2}{L} \frac{l_b^2}{(2\pi f)^2} \]  
\[ (2.4.7) \]
Table III.23 below summarizes the typical values for ATM band-limited exponential parameters retrieved over different sea ice scenarios:\footnote{Some flights are excluded from the table, either because they had no ATM LIDAR data (C,G) or many missing samples (D,F)}:

Table III.23 - Exponential parameters for typical arctic surfaces from ATM data:

Band-limited root mean square height $\sigma_b$ and correlation length $l_b$

<table>
<thead>
<tr>
<th>Bandpass $\Delta x / L$ (m)</th>
<th>$f_{\text{min}} / f_{\text{max}}$ (m$^{-1}$)</th>
<th>$\sigma_b$ (cm)</th>
<th>$l_b$ (m)</th>
<th>Arctic region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young ice</td>
<td>8 / 100</td>
<td>0.015 / 0.0625</td>
<td>5-10</td>
<td>2-5</td>
</tr>
<tr>
<td>Level FY</td>
<td>5-10</td>
<td>5-6</td>
<td>A2, A3 (Point Barrow)</td>
<td></td>
</tr>
<tr>
<td>Lightly rough FY</td>
<td>10-20</td>
<td>5-9</td>
<td>B4, B5 (Beaufort Sea)</td>
<td></td>
</tr>
<tr>
<td>Deformed FY</td>
<td>20-35</td>
<td>6-10</td>
<td>A1, A4 (Point Barrow)</td>
<td></td>
</tr>
<tr>
<td>Multiyear</td>
<td>15-25</td>
<td>6-10</td>
<td>B1, B2 (Beaufort Sea)</td>
<td></td>
</tr>
</tbody>
</table>

The exponential parameters, $\sigma_b$ and $l_b$, are band-limited in that they are in principle only applicable within the profiler sampling bandpass. Before we attempt to extrapolate these band-limited models outside of their sampling passbands, we must ask how far up in frequency we can push the models before they start deviating from reality. For this purpose, we compare the ATM model extrapolations against sea ice roughness descriptors reported by other investigators for smaller length scales.

Table III.24 - Exponential parameters for typical arctic surfaces in various spectral bands, from independent sources.

<table>
<thead>
<tr>
<th>Bandpass $\Delta x / L$ (m)</th>
<th>$f_{\text{min}} / f_{\text{max}}$ (m$^{-1}$)</th>
<th>$\sigma_e$ (cm)</th>
<th>$l_e$ (m)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young ice</td>
<td>0.05 / 20</td>
<td>0.05 / 10</td>
<td>5</td>
<td>1.5</td>
</tr>
<tr>
<td>Level ice</td>
<td>0.002 / 1</td>
<td>1 / 250</td>
<td>0.5</td>
<td>0.07</td>
</tr>
<tr>
<td>Deformed ice</td>
<td>0.05 / 10</td>
<td>0.1 / 10</td>
<td>14</td>
<td>1.20</td>
</tr>
<tr>
<td>Multiyear ice</td>
<td>0.0005 / 1</td>
<td>1 / 1000</td>
<td>0.5</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table III.24 above summarizes the exponential parameters for typical arctic surfaces observed by various independent sources. The definition of the sampling bandpass

\footnote{Some flights are excluded from the table, either because they had no ATM LIDAR data (C,G) or many missing samples (D,F)}
The applicable to these attributes is expressed in the first column in terms of sampling spacing \( \Delta x \) and profile length \( L \) found in the reference cited. These attributes allow us to reconstruct the spectral behavior of sea ice roughness using lorentzian curves within the spectral regions in which each source operates. The information provided in Table III.24 is laid out graphically in Figure III.23 (dashed lines), along with the typical roughness levels (observed in black and extrapolated in red) measured by the ATM profiler.

\[
S(k) = 2\sigma_b^2 l_b /[1 + (l_b k)^2]
\]

and extrapolated behavior (in red) versus independent data (dashed lines)

Based on these graphs, we can infer that the ATM lorentzian curves conform to independent spectral roughness data up to frequencies of about 1-10 m\(^{-1}\) (i.e. see the young and deformed ice cases), and that they overestimate by about an order of magnitude the roughness typically observed at scales beyond that limit (e.g. at the centimetric and millimetric scales, especially in the multiyear ice case), probably due to a transition into a different roughness regime that cannot be accounted for by meter-scale ATM roughness parameters. In the light of these comparisons, the range of spatial
frequencies over which the ATM LIDAR exponential parameters provide a realistic representation of the state of the “large scale” sea ice roughness is defined, leaving us in position to calculate sea ice roughness RMS height/slope statistics of variable spectral location and bandwidth within those limits.

**Large scale roughness**

The table III.25 below shows the radiometric class clusters\(^4\) from Section 2.3, ordered according to their stage of development (i.e. proxy thickness) with the corresponding “large scale” roughness descriptors from the LIDAR profiler, mean square slope (\(mss\)) and root mean square height, integrated across a passband that corresponds to a virtual sampling distance of 1 meter and virtual profile length of 100 meters.

<table>
<thead>
<tr>
<th>Class</th>
<th>LABEL</th>
<th>mss</th>
<th>RMS (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MYI</td>
<td>B1</td>
<td>MYI</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>MYI</td>
<td>0.008</td>
</tr>
<tr>
<td>Shorefast FYI</td>
<td>A1</td>
<td>Shorefast deformed FYI</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>Shorefast FYI</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>Shorefast FYI</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>F4</td>
<td>Shorefast FYI</td>
<td>0.008</td>
</tr>
<tr>
<td>FYI</td>
<td>B3</td>
<td>Deformed FYI</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>B5</td>
<td>FYI</td>
<td>0.006</td>
</tr>
<tr>
<td>Thin FYI</td>
<td>A4</td>
<td>Deformed Thin FYI</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>B4</td>
<td>Thin FYI</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>F5</td>
<td>Thin FYI</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>D4</td>
<td>Thin FYI</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>Thin FYI</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>Thin FYI</td>
<td>0.004</td>
</tr>
<tr>
<td>YI/NI</td>
<td>E3</td>
<td>New-young ice</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>E2</td>
<td>New ice</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>E1</td>
<td>New ice</td>
<td>0.003</td>
</tr>
</tbody>
</table>

\(^4\) Classes F1, F2, F3, D3 and all C and G classes have been excluded from this table, because they are either not covered by ATM data or had too few samples (< 150) to be statistically significant.
Figure III.24 – Passive microwave, SAR backscatter and LIDAR large scale roughness signatures
The information provided by the LIDAR profiler allows us to inquire about the impact of large scale roughness on the microwave signatures of sea ice. Figures III.24 (a)-(d) above summarize the polarization, spectral gradient, SAR backscatter and large scale roughness signatures of a set of labeled sea ice clusters, ordered following their proxy thickness, from multiyear (Class B1) to new ice (Class E1). Note that the utilization of the PR-GR scatter plot in Figure III.20 is justified by the large correlation between the polarization and spectral gradient ratios calculated at different frequencies.

A clear feature in Figure III.24 (a) is the strong decrease in polarization from maximum values for new ice to minimum values for thin ice (Classes E1 through A4), followed by a small increase in polarization as sea ice develops further from first year ice into multiyear. In contrast, the spectral gradient signature in Figure III.24 (b) decreases rather monotonically with sea ice thickness, with exception made for the brine layered thin ice types (Classes D1 to D4). Observe that the variability in passive microwave signatures becomes weaker for thin FYI, FYI and MYI types, reflecting a problematic ambiguity in the radiometric identification of these types, as already mentioned (see Figure III.20). However, the variability in the large scale roughness signature shown in Figure III.24 (d) appears largest in the Thin FYI to MY range, suggesting that the monitoring of this parameter may help augment the existing sea ice classification algorithms. After appropriate masking of land-bound sea ice classes (i.e. those sea ice forms whose roughness is determined by the presence of continental accidents, forming either ultra-smooth land-protected sea ice forms or ultra-rough land-crushed surfaces, like Classes A1, A2, A3 and A4 in Barrow), the large scale mean square slope is found to increase with sea ice thickness. The fact that neither the polarization nor the spectral gradient ratio correlate with the presence of the land-bound classes is indicative of a general lack of response of radiometric signatures to large scale roughness. On the other hand, the SAR backscatter signature in Figure III.24 (c) is sensitive to the presence of outlying land-bound classes. After appropriate masking of these land-bound types, the SAR backscatter
signature follows an already documented behavior, namely a decrease in going from MYI to FYI followed by an increase for saline thin FYI/YI types and back down to noise-level for new ice [Onstott, 1992].

3 Bistatic model inversion

![Diagram of model inversion strategy]

In this section, sea ice parameters such as mean surface height, roughness and dielectric permittivity are derived from scattered GPS waveforms. Our overall strategy is presented in Figure III.25, where scattered GPS waveforms are to be fit to a forward model constructed using either the KAGO or SSA scattering cross sections. The model fit is based on a computational search for least square residuals between measured and model GPS waveforms, the latter calculated for a discrete set of surface states [Bevington & Robinson, 2003], and the inversion products are compared against external references to evaluate the goodness of the fit.

The forward model waveforms are calculated as:

\[
\chi_{\text{model}}^n (\tau, \omega) = \int_{-\infty}^{\infty} \frac{\sigma^0 (\bar{\rho}) \chi(\tau, \omega; \bar{\rho})}{4\pi R^2 (\bar{\rho})} d^2 \rho
\]  

(3.1)
Where (see Section 2 in Chapter I),
\[
\chi(\tau, \Delta \omega) = \left| \Lambda_{PRN}(\tau) \right|^2 \sin(\Delta \omega \tau_{int} / 2) \left| \Delta \omega \tau_{int} / 2 \right|^2 \approx \left| \Lambda_{PRN}(\tau) \right|^2
\]  
(3.2)

That is, we assume that the Doppler spread in scattered signals due to the receiver motion is negligible (i.e. that most surface contributions have frequency shifts \(\Delta \omega/2 \pi << 1/\tau_{int} = 1\) KHz). The radar cross sections (coherent + diffuse) for the KAGO and SSA models are expressed as:
\[
\sigma^0_{LR}(mss)_{KAGO} = \pi \left( \frac{kq}{q_z^2} \right)^2 \left| C_{LR} \right|^2 \int_{\mathbb{F}_{\mathbb{V}_z}(\theta)} PDF_{\mathbb{V}_z}(\theta) d^2 x
\]
(3.3)

\[
\sigma^0_{LR}(\sigma, l)_{SSA} = \pi \left( \frac{k^2}{2\pi q_z} \right)^2 \left| e^{-1} f_{LR} \right|^2 \int e^{i\theta} e^{-q^2 \sigma_z^2 (1-\rho)^2} d^2 x
\]
(3.4)

Where (see Appendices G and H),
\[
C_{LR} = \frac{1}{2} (C_{hh} - C_{vv}) \quad f_{LR} = \frac{1}{2} (f_{hh} - f_{vv} + if_{hv} + f_{vh})
\]
(3.5)

The surface autocorrelation function of sea ice heights (for the SSA model) and the PDF of slopes (for the KAGO model) are assumed the following:
\[
\rho(r; \sigma, L_1, L_2) = e^{-r^2/(2L_1^2)}
\]
(3.6)

\[
PDF_{\mathbb{V}_z}(\theta) = \frac{1}{2\pi mss} e^{-\frac{(q_z/q)^2}{2mss}} \quad \text{with} \quad mss = 2\sigma^2 / L_2 L_3
\]
(3.7)

This choice ensures that surface roughness remains exponentially correlated at large scales. Our preliminary analysis of sea ice roughness using the LIDAR profiler (see Section 2.4) suggests that the scale parameter \(L_2\), which controls the onset of spectral dampening of roughness at small wavenumbers, be set at about 0.5 m in order to reproduce a decay of about an order of magnitude in the 10-100 m\(^{-1}\) wavenumber range (see Figure III.23). To simplify the inversion procedure, the exponential correlation length \(L_1\) in (3.6) has been initially fixed to a typical value of 5.5 m (see Table III.23), leaving the surface RMS height (or equivalently, the surface mean square slope) as the only free
parameter in the model inversion, which is then based on the minimization of the cost function $J(\tau, \text{mss})$:

$$J(\tau, \text{mss}) = \int \left| \chi_{\text{scat}}^{\text{model}}(\tau' - \tau, \text{mss}) - \chi_{\text{scat}}^{\text{measure}}(\tau') \right|^2 d\tau'$$

Where $\tau$ is the scattered signal delay. To eliminate the additional degree of freedom associated with changes in surface reflectivity, which do not appreciably modify the shape of the waveforms (see Figure II.16 in Chapter II), both model and measured waveforms are normalized to unit peak power before they enter the minimization process. Table III.26 shows the bounds on model parameters and selection of steps sizes in the discretization of the forward model search space, selected to keep differences between model waveforms below measurement noise levels (-22 dB in terms of effective GPS reflectivity, see Section 3.1.2.1).

| Table III.26 – Forward model search space: bounds on parameters and step sizes |
|-----------------|-----------------|-----------------|
| Low Altitude    | High Altitude   |                 |
| Receiver height | 100 : 50 : 300 m| 1160 : 80 : 1240 m|
| Elevation       | 25° : 5° : 65°  |                 |
| $\text{mss}$    | 0.002 : 0.004 : 0.050 |               |

Every model waveform carries some information about the scattering cross-section of the surface, $\sigma^0$. The leading edge of the waveform collects power contributions from surface returns that are closer to the specular point, and thus bears upon the specular lobe of the cross-section, while the trailing edge of the waveform collects contributions from points further off the specular point, probing into the off-specular and Bragg/resonant tails of the surface cross-section.

Figure III.26 shows sample model waveforms that arise using the KAGO and SSA cross sections for different states of surface roughness. Since, for a given incidence angle, the SSA cross-section decays more sharply than the KAGO cross-section (see Figure II.15 in
Chapter II), SSA waveforms feature somewhat earlier leading edges and are more narrow than KAGO’s. The effect of Bragg/resonant scattering from small scale roughness in SSA waveforms appear as a rise in the waveform tails well beyond the first few chips, however well under typical noise levels. Thus the main difference between KAGO and SSA waveforms lies in their different treatment of near-specular scattering, or the number of terms used to expand the surface autocorrelation function about small surface displacements.

Figure III.26 – Forward model waveforms: receiver altitude is 1200 m and satellite elevation is 65 deg (normalized to unit peak amplitude, mss = 0 to 0.05)

3.1 KAGO/SSA models

This section summarizes the products that result after inversion of the KAGO and SSA models above with the surface constraints already introduced, namely $L_1 = 5.5$ m and $L_2 = 0.5$ m. These assumptions, based on coincident observations of the air-snow interface using a LIDAR profiler, are revised and modified as comparisons with other reference data are carried out.

3.1.1 Altimetry

Early studies have pointed at difficulties in the simultaneous extraction of both signal delay and surface roughness from scattered GPS waveforms collected at airborne
altitudes [Rius et al., 2002]. We observe that the determination of surface topography and roughness using scattered GPS waveforms is hampered by a parameter degeneracy in the delay/roughness search space (see Figure III.27). The parameter degeneracy, which causes rough model waveforms to look like delayed versions of smoother model waveforms, results in a family of model delay/roughness pairs that yields minimum residuals when fit to observed waveforms. Most frequently, the parameter degeneracy leads to an overestimation of roughness, which subsequently leads to negatively biased surface heights (i.e. shorter model delays), with altimetry errors on the order of tens of meters (see left panel in Figure III.28).

Figure III.27 – Parameter degeneracy in search space: cost function $J(\tau, mss)$: minimum residuals are distributed along a family of delay/roughness pairs

a) Low altitude ($h=200$ m)  
b) High altitude ($h=1200$ m)

Figure III.28 – Roughness induced biases in altimetry determination: the overestimation of roughness (blue lines in lower plots) induces negative height biases (blue lines in upper plots). Better roughness retrievals require externally constrained signal delays (red lines)
Although the parameter degeneracy is somewhat relieved at higher receiver altitudes (see right panel in Figure III.28), the determination of the sea ice topography using scattered signals is overall poor compared with that provided by the LIDAR, as shown in Figure III.29a, with elevation and model dependent biases on the order of a few meters, and standard deviations of the same order. Under these circumstances, it is difficult to compare the performance of the KAGO and SSA models. Suffice it to say that both models agree best over smooth patches of highly reflective new ice (i.e. for quasi-specular waveforms in St Lawrence, see Figure III.29b), which we have used in determination of the instrument line bias (i.e. electronic delay = -13 m). Since the delay and roughness parameters are degenerate in scattered GPS signals, a poor determination of the instrument line bias (i.e. badly constrained signal delays) may lead to strong biases in roughness estimates, especially at low receiver altitudes.

3.1.2 Roughness and permittivity

Because the simultaneous determination of roughness and signal delay estimates from scattered GPS waveforms shows signs of degeneracy, the inversion of surface roughness that follows is performed after constraining the signal delay externally using the LIDAR reference heights, providing for a correction of the instrumental line bias.
**Permittivity**

An effective GPS reflectivity \( R_{GPS} \) is calculated as:

\[
R_{GPS} = \left| \mathcal{R}_{\text{eff}} \right|^2 = \frac{3}{2T_c} \int \mathcal{X}_{\text{scat}}(\tau_r)d\tau_r
\]  

(3.1.2.1)

The effective GPS reflectivity \( R_{GPS} \) is mapped to an effective (roughness and geometry independent) L-band permittivity \( \varepsilon_{\text{eff}} \) via the Fresnel reflection coefficients (see Figure III.30):

\[
R_{GPS}(\theta_i) = \frac{1}{4} \left| \mathcal{R}_{\text{hh}}(\theta_i) - \mathcal{R}_{\text{vv}}(\theta_i) \right|^2 = \frac{1}{4} \cos \theta_i - \sqrt{\varepsilon_{\text{eff}} - \sin^2 \theta_i} - \varepsilon_{\text{eff}} \cos \theta_i - \sqrt{\varepsilon_{\text{eff}} - \sin^2 \theta_i}
\]

The lack of a stored record of the AGC operation during data collection (see Section 3.3 in Chapter I) forces us to assume that the L-Band (receiver+scene) input noise levels into

---

5 The reflectivity is proportional to the area under the scattered waveform. The \( 2T_c/3 \) factor is the area of the undistorted GPS autocorrelation function \( \chi \):

\[
\int \mathcal{X}_{\text{scat}}(\tau_r)d\tau_r = \frac{\sigma^0(\bar{\rho})}{4\pi R^2(\bar{\rho})} \left( \int \chi(\tau_r, \tau(\bar{\rho}))d\tau \right) d^2\rho = \frac{2T_c}{3} \left( \frac{\mathcal{E}_{\text{scat}}}{E_{\text{inc}}^2} \right)_{\text{surf}} = \frac{2T_c}{3} \left| \mathcal{R}_{\text{eff}} \right|^2
\]
the GPS radar stay the same for uplooking and downlooking channels, keeping the ratio $\alpha_{\text{AGC}} = \text{AGC}^{\text{up}}/\text{AGC}^{\text{down}}$ a constant. In order to match a dielectric permittivity of 80 over open water surfaces, the scattered waveforms are calibrated with a factor $\alpha_{\text{AGC}} = 2$, in rough correspondence with a downlooking channel that sees about twice as much noise as the uplooking channel (for an antenna noise temperature of $\sim$200 K, a negligible L-band sky temperature and L-band thermal emissions $\sim$200 K for sea ice [Grenfell et al., 1998]).

**Surface roughness**

The correspondence between surface mean square slopes derived from the model inversion and surface RMS heights is dependent on the assumptions made about the surface roughness spectrum via the expression [see (3.7)]

$$\sigma^2 = \frac{L_1 L_2}{2} \sigma_{\text{mss}}$$

The surface parameters $L_1$ and $L_2$ are built in within the SSA model (that is, any change in these parameters affects the shape of the forward model waveforms), but the KAGO model offers a larger degree of freedom, in that $L_1$ and $L_2$ remain undetermined, as long as their product is greater than the wavelength squared (to satisfy the requirement of small surface curvature). Figure III.31 shows the distribution of observed surface RMS heights with signal incidence angle, according to the KAGO and SSA model inversions, with LIDAR estimates as a reference. The LIDAR reference RMS heights are integrated over a bandpass that contains spatial scales between 1-100 m, in rough correspondence with the expected spatial bandpass of scattered GPS power (which tentatively includes scales that range from the radiation wavelength $\lambda_{\text{GPS}} = 0.2$ m to the correlation width of the replica on the surface $r_{\text{CA}} = 500$ m).
Figure III.31 – Model and observed surface roughness (RMS height): all flights

Note that the LIDAR RMS heights are about 2 times larger than either KAGO or SSA estimates, and that while LIDAR RMS heights do not show any particular trend, SSA roughness estimates tend to increase with incidence angle. Because of the inherent freedom in the choice of surface parameters $L_1$ and $L_2$ for KAGO retrievals, we find that scaling the KAGO $mss$ products by a factor of 4 provides a better match with the LIDAR reference. This scaling may be absorbed by new effective surface parameters such that:

$$(L_1L_2)_{\text{effective}} = 4L_1L_2$$

Which corresponds to a gaussian correlation length of 3.3 m. This modification admits a limited number of interpretations (see Figure III.32): either the scale $L_2$ of onset of spectral decay of roughness is larger than we assumed (SSA1), the effective surface exponential correlation length is larger than we assumed (SSA2), or a combination of both (SSA3). In any case, it seems that our current surface model contains too much small scale roughness, which increases the width of the waveforms without calling for increased surface RMS heights. Changing the surface parameters $L_1$ and $L_2$ entails the formation of a new database of SSA model waveforms and new inversion retrievals. As shown in Figure III.33, we observe that GPS altimetry biases are especially sensitive to the spectral dampening scale $L_2$ of the surface (i.e. the presence of small scale roughness),
whereas GPS roughness RMS height estimates are more sensitive to the correlation length $L_1$ of the surface (i.e. the presence of large scale roughness). An optimal match in Figure III.33 of the SSA model altimetry and roughness products to the LIDAR reference seems to call for an increase in $L_1$ and $L_2$ (i.e. a net decrease of small scale roughness) with incidence angle.

Figure III.32 – Effect of new surface parameters in KAGO (red) and SSA (black) cross-sections at 90 deg (top row) and 30 deg (bottom row) satellite elevations. LR polarization and RMS height = 0.25 m

Figure III.33 – Effect of new effective surface parameters in altimetry and roughness retrievals (SSA0, SSA1, SSA2 and SSA3)
Although the SSA model allows the formulation of more elaborate questions about the spectrum of surface roughness and its role in the scattering process, we find that the number of uncorrelated measurements in scattered waveforms (3-4 samples at altitudes of about 1000 m for single satellite observations, see Figure I.9) conditions the number of surface parameters that can be observed independently. To further on the application of the SSA model to GPS bistatic scattering, we would recommend either flying at higher altitudes or using a GPS code with a shorter correlation width (such as the GPS P code, [Navstar, 1991]).

Shown next are our final KAGO model inversion products, dielectric permittivity and surface RMS height, obtained using externally constrained signal delays and the power/correlation length calibration just introduced. For reference purposes, the satellite elevation angle and the LIDAR surface RMS heights are also shown.
Figure III.34 – Barrow flight (high altitude, \( h=1200 \) m)
Figure III.35 – Barrow flight (low altitude, \( h=200 \) m)
a) elevation (↑) and GPS permittivity (↓)  
b) LIDAR (↑) and GPS (↓) RMS height  

c) LIDAR roughness  
d) GPS roughness  

e) GPS permittivity  

Figure III.36 – Icecamp flight (high altitude, h=1200 m)
a) elevation (↑) and GPS permittivity (↓)  
b) LIDAR (↑) and GPS (↓) RMS height

d) GPS roughness

e) GPS permittivity

Figure III.37 – St Matthew flight (high altitude, \( h = 1200 \) m)
Figure III.38 – Norton Sound flight (high altitude, \( h = 1200 \) m)

a) elevation (↑) and GPS permittivity (↓)  
b) LIDAR (↑) and GPS (↓) RMS height  
c) LIDAR roughness  
d) GPS roughness  
e) GPS permittivity
Figure III.39 – St Lawrence flight (low altitude, \( h=200 \text{ m} \))
Figure III.40 – Point Hope flight (high altitude, $h=1200$ m)
Figure III.41 – Bering flight (high altitude, $h=1200$ m)
We observe in Figures III.34 through III.41 above that KAGO roughness estimates are in general good agreement with their LIDAR counterparts. Localized differences are to be attributed mainly to different penetration depths, and to a lesser extent to distant footprints, which may be as far as 2 km apart at low GPS signal elevations (see Figure III.5).

While LIDAR returns come from the smoothed out air-snow interface, the snow cover is transparent to GPS signals, which penetrate down to the underlying snow-ice or snow-land interface. Observe, for instance, the GPS roughness estimates from our high altitude flight over Barrow (Figure III.34): while the LIDAR profiler does not sense an appreciable amount of roughness over the land areas (at the start and end tails of the time series), the GPS sensor shows a significant response. The GPS sensor also responds relatively more strongly to the striated band of shear/pressure ice north of Point Barrow (the radiometric class A1, during the fourth flight segment) than the LIDAR sensor. Observe too that GPS roughness seem to correlate better with the snow-covered class A2 in the SE end of Elson Lagoon, suggesting that some underlying roughness is responsible for the accumulation of snow drifts in that area. In our low altitude flights over Barrow (see Figure III.35) we note that, although large scale features in the striated shear/pressure zone are correctly represented, KAGO GPS estimates have a lower degree of agreement with LIDAR estimates, a fact that is connected to a greater sensitivity of GPS roughness estimates to errors in the externally constrained signal delays at low receiver altitudes.

The flight over the Icecamp in the Beaufort Sea (see Figure III.36) demonstrates the potential of scattered GPS signals for the detection of multiyear ice via measurements of large scale roughness. The agreement with LIDAR roughness in this flight is good and the presence of multiyear floes (classes B1 and B2) clearly revealed in an increase of surface RMS heights. The flight over St Lawrence (see Figure III.39) illustrates another salient characteristic of our GPS roughness retrievals. The KAGO RMS heights show a
lower saturation limit of about 10 cms, near the Rayleigh limit for smooth surfaces, beyond which scattered waveforms become specular and no longer carry information about surface roughness. The limited usefulness of scattered signals in measuring roughness over smooth surfaces (i.e. over new and young ice types) is counterbalanced by a deeper penetration and larger permittivity response in that regime. The flight over Point Hope (see Figure III.40) provides a beautiful example of how GPS L-band permittivities can be used, not only to delineate the sea ice edge (see St Matthew and Bering flights in Figures III.37 and III.41), but also to map thin ice types. Indeed, the GPS bistatic radar works as a precise L-band radiometer that directly measures the surface emissivity (via surface reflectivities), bypassing the troublesome inference of the emitting layer physical temperature inherent in true radiometer measurements.

4 Discussion

To evaluate the overall usefulness of GPS roughness and L-band permittivities inferred from scattered signals using the KAGO scattering model, we re-examine the separability of the radiometric sea ice groups introduced in Section 2.3 in terms of the new GPS retrievals. The lower frequency (deeper penetration depth) of GPS signals should in principle afford a better separation of thin ice types, while the signal responsiveness to multiyear ice in terms of surface roughness should allow for a better separation of thick ice types. Table III.27 and Figures III.42 through III.44 provide a quantitative and graphical summary of the radiometric and GPS signatures of identified sea ice clusters observed during the AMSRIce03 validation campaign.

As we have already mentioned, the separability of sea ice clusters based on their radiometric signatures at 11, 19 and 37 GHz alone is acceptably good for thin ice types (open water, new ice and young ice), but the identification of thicker ice types (thin first
year, first year and multiyear ice) could only be afforded after a considerable contextual interpretation effort that included SAR and visible imagery, along with geographical constraints. Figure III.43 shows proof of this difficulty in the large degree of radiometric overlap for the thick ice classes. Although radar backscatter can be used to help discriminate multiyear from first year ice types, we find that the backscatter signature of thinner but more saline sea ice types can be easily misinterpreted as that of thick multiyear ice (Figure III.42E).

Table III.27 – Summary of radiometric and GPS signatures of labeled sea ice clusters

<table>
<thead>
<tr>
<th>GROUP</th>
<th>CLASS</th>
<th>GPS Permittivity</th>
<th>GPS RMS height (cm)</th>
<th>LIDAR RMS height (cm)</th>
<th>PR_{19V}</th>
<th>GR_{19,37V}</th>
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<tbody>
<tr>
<td>G1</td>
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<td>0.06</td>
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<tr>
<td>G2</td>
<td>OW</td>
<td>31.8</td>
<td>11</td>
<td>-</td>
<td>0.16</td>
<td>0.03</td>
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<tr>
<td>C1</td>
<td>OW</td>
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<td>14</td>
<td>-</td>
<td>0.21</td>
<td>0.05</td>
</tr>
<tr>
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<td>OW</td>
<td>20.5</td>
<td>11</td>
<td>6</td>
<td>0.23</td>
<td>0.04</td>
</tr>
<tr>
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<td>OW</td>
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<td>12</td>
<td>-</td>
<td>0.19</td>
<td>0.02</td>
</tr>
<tr>
<td>E2</td>
<td>NI</td>
<td>13.5</td>
<td>11</td>
<td>7</td>
<td>0.15</td>
<td>0.02</td>
</tr>
<tr>
<td>G3</td>
<td>NI</td>
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<td>11</td>
<td>-</td>
<td>0.11</td>
<td>0.02</td>
</tr>
<tr>
<td>E3</td>
<td>NI</td>
<td>11.2</td>
<td>11</td>
<td>7</td>
<td>0.12</td>
<td>0</td>
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<td>NI</td>
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<td>11</td>
<td>-</td>
<td>0.17</td>
<td>0.02</td>
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<td>NI</td>
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<tr>
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<td>-</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
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<td>YI</td>
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<td>0</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
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<td>-</td>
<td>0.06</td>
<td>0</td>
</tr>
<tr>
<td>D3</td>
<td>YI</td>
<td>3.6</td>
<td>14</td>
<td>-</td>
<td>0.09</td>
<td>-0.01</td>
</tr>
<tr>
<td>D2</td>
<td>Th FYI</td>
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<td>14</td>
<td>16</td>
<td>0.07</td>
<td>-0.04</td>
</tr>
<tr>
<td>F5</td>
<td>Th FYI</td>
<td>2.8</td>
<td>15</td>
<td>17</td>
<td>0.03</td>
<td>0</td>
</tr>
<tr>
<td>D1</td>
<td>Th FYI</td>
<td>2.8</td>
<td>14</td>
<td>16</td>
<td>0.09</td>
<td>-0.06</td>
</tr>
<tr>
<td>D4</td>
<td>Th FYI</td>
<td>2.6</td>
<td>16</td>
<td>17</td>
<td>0.06</td>
<td>-0.03</td>
</tr>
<tr>
<td>C5</td>
<td>Th FYI</td>
<td>2.5</td>
<td>12</td>
<td>-</td>
<td>0.05</td>
<td>-0.01</td>
</tr>
<tr>
<td>A4</td>
<td>Th FYI (Rough)</td>
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<td>19</td>
<td>27</td>
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<td>0</td>
</tr>
<tr>
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<td>2.2</td>
<td>14</td>
<td>14</td>
<td>0.04</td>
<td>-0.01</td>
</tr>
<tr>
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<td>FYI (Fast)</td>
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<td>16</td>
<td>20</td>
<td>0.04</td>
<td>-0.01</td>
</tr>
<tr>
<td>A3</td>
<td>FYI (Fast)</td>
<td>2.2</td>
<td>13</td>
<td>8</td>
<td>0.04</td>
<td>-0.01</td>
</tr>
<tr>
<td>B5</td>
<td>FYI</td>
<td>2.1</td>
<td>14</td>
<td>15</td>
<td>0.03</td>
<td>-0.01</td>
</tr>
<tr>
<td>B3</td>
<td>FYI (Rough)</td>
<td>2.0</td>
<td>16</td>
<td>17</td>
<td>0.04</td>
<td>-0.02</td>
</tr>
<tr>
<td>A2</td>
<td>FYI (Fast)</td>
<td>2.0</td>
<td>15</td>
<td>8</td>
<td>0.04</td>
<td>-0.02</td>
</tr>
<tr>
<td>A1</td>
<td>FYI (Fast rough)</td>
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<td>26</td>
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<td>-0.02</td>
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<tr>
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<td>1.9</td>
<td>18</td>
<td>18</td>
<td>0.04</td>
<td>-0.02</td>
</tr>
<tr>
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<td>1.8</td>
<td>20</td>
<td>20</td>
<td>0.05</td>
<td>-0.03</td>
</tr>
</tbody>
</table>
Figure III.42 - Summary of radiometric and GPS signatures of labeled sea ice clusters
Figure III.43 - Radiometric separability of AMSRIce03 ice clusters

Figure III.44 - GPS separability of AMSRIce03 ice clusters

Figure III.44 shows the distribution of AMSRIce03 sea ice clusters in the space of GPS retrievals. The separability of thin ice types in terms of the surface permittivity inferred from scattered GPS signals is as good as that provided by the PSR multi-frequency polarimetric radiometer, showing a strong decrease from nearly 100 to about 5 as sea ice evolves from open water into thin first year. Observe too that whereas new ice is
characterized by its smoothness (in that it lies right at the lower saturation limit for GPS roughness), open water has a more variable roughness signature (typically wind dependent). The ordering of thicker ice types in terms of GPS permittivity is consistent but affords a less clear separability, with values ranging from about 5 for thin first year ice to 1.8 for multiyear. This loss in resolution in terms of GPS permittivity is somewhat compensated by an increased separation in terms of surface roughness, particularly between first year and multiyear ice. Unfortunately, the overlap between thin first year and first year ice classes in terms of GPS retrievals remains problematic, and as we found in our preliminary evaluation of LIDAR roughness, the separability of multiyear from first year ice relies on an efficient masking of the “ultra-rough” land-bound sea ice clusters near continental obstacles (such as A1 and A4 in Point Barrow).

In any case, additional observations of surface L-band permittivity and large scale roughness provided by the GPS bistatic radar should allow for a better understanding of the intrinsic variability in radiometric signatures (i.e. polarization and spectral gradient ratios) across sea ice classes.
CHAPTER IV – Conclusion
Summary, conclusion and future work

Although the combined active and passive satellite microwave record over the period 1979 to 2006 indicates that the Arctic sea ice extent has declined for every month, evidence for accompanying reductions in ice thickness is inconclusive ([Stroeve et al., 2005], [Kwok, 2007]). Our study of the distribution of sea ice types during the Arctic spring of 2003 over the Bering, Chukchi and Beaufort Seas, which include multiyear and thick first year ice, as well as areas of sea ice formation and diffuse sea ice margins, shows that passive microwave measurements are able to identify sea ice types with thicknesses below 30 cm unambiguously, but they offer a poorer resolution of older ice types, whose thicknesses range from 30 to 400 m. This lower resolution in the thick ice range is partially overcome by observations of radar backscatter, which prove very sensitive to the presence of multiyear ice via enhanced volume scattering, but that nonetheless remain ambiguous when thinner, rougher and more saline ice types are present.

In this context, the GPS bistatic radar proves able to function as both an L-band radiometer and a LIDAR profiler simultaneously. The retrieval of sea ice surface roughness and permittivity values from scattered GPS signals is enabled by the inversion of a forward model that encodes the scattering cross-section of the surface. The resulting GPS roughness estimates are in good agreement with those provided by the reference LIDAR profiler, and measured L-band permittivities distribute consistently across identified sea ice clusters that represent different levels of ice thickness. The simultaneous retrieval of roughness and permittivities using scattered GPS signals allows breaking the salinity/roughness ambiguity inherent to radar backscatter measurements and provides an improved separability of thick sea ice classes by the addition of a large-scale surface roughness observable.
The application of a simple electromagnetic Kirchhoff model proves sufficient for the retrieval of surface parameters using scattered GPS signals collected at airborne altitudes, although the technique remains suboptimal for precision altimetry, in the sense that scattered waveforms cannot attain the centimeter-accuracy required to measure the ice free-board height above the sea level. The utilization of a more elaborate Small Slope Approximation model, which should afford a more complete description of the sea ice surface and better corrections for altimetry biases, is made difficult by an insufficient number of uncorrelated power samples in waveforms and the lack of a good a priori model for the roughness spectrum, which renders the problem of inversion of surface parameters ill conditioned. While these conclusions apply in principle to GPS signals scattered from sea ice, they should hold as well for GPS signals scattered from soil or ocean surfaces.

Some suggestions for future work include the direct validation of the sea ice roughness/thickness relationship using *in situ* measurements in the Arctic, and an extended application of the GPS radar technique to Antarctic sea ice, where different conditions may prevail. In order to fully exploit the capabilities of the bistatic technique, we recommend to perform an analysis of GPS waveforms either collected at higher altitudes, or using a larger bandwidth code (such as the GPS P code), using the SSA scattering model. Under these conditions, we would expect that the degeneracy that prevents the simultaneous retrieval of signal delays and surface roughness will be broken (rendering the bistatic technique more autonomous), and that waveforms will provide a better sampling of the angular pattern surface cross-section, which could help solve a number of questions relative to the ability of the scattering model to effectively map the surface autocorrelation function, and about the potential band-dependence of the resulting surface parameters with changing incidence angles.
References and Appendices
Bibliographical references


Appendix A – Radiometric brightness temperature

At long wavelengths, in the Rayleigh-Jeans approximation of Planck’s law for bodies in thermal equilibrium with their environment, the radiant emittance of a graybody with emissivity \( \varepsilon(\theta) \) is given by:

\[
E(\nu) = B(\nu) \cdot \varepsilon(\theta) \cdot d\Omega' \cdot ds \quad \text{(W/Hz)}
\]

\[
B(\nu) = 2k_B T(\nu/c)^2 \quad \text{(W/m}^2\text{Hz sr)}
\]

Where \( B(\nu) \) is the isotropic blackbody surface radiance. The differential power collected by a linearly polarized antenna of effective area \( A_e(\theta, \phi) \) at a distance \( R \) from a surface element of area \( ds \) in a spectral band \( d\nu \) is given by:

\[
dP_r = \left( \frac{dE}{d\Omega'} \right) d\Omega_{\text{detector}} d\nu_{\text{detector}} = \frac{1}{2} B(\nu) \varepsilon(\theta) ds \left( \frac{A_e(\theta, \phi)}{R^2} \right) \varepsilon(\theta) ds \left( \frac{A_e(\theta, \phi)}{A^2} \right) = k_B T(\theta) \varepsilon(\theta) d\Omega \left( \frac{G(\theta, \phi)}{4\pi} \right)
\]

The total received power in a spectral band-channel is then given by:

\[
P_r = k_B \left( \frac{1}{4\pi} \int_{\Omega_{\text{sky}}} T(\theta) G(\theta, \phi) d\Omega \right) \Delta\nu_{\text{detector}} \equiv k_B T_b \Delta \nu_{\text{detector}}
\]

The brightness temperature \( T_b \) observed by the radiometer is equal to the average physical surface temperature weighted by both the surface angular emissivity and the receiving antenna gain pattern \( G(\theta, \phi) \). The measured value of \( T_b \) remains strongly dependent on the observation angle, the polarization, the surface roughness and the effects of atmospheric emission and absorption. In deriving this formula, it is assumed that the atmosphere is transparent and that the sky is cold, radiometrically speaking.

---

6 Emittance = radiant power per unit bandwidth from a surface element \( ds \) into a solid angle \( d\Omega' \)
Appendix B – Antenna pattern correction

The antenna pattern correction factor $G_{\text{up}}/G_{\text{down}}$ in (3.3.4) of Chapter I requires knowledge of the receiving antenna gain patterns and their relative orientation. The GPS antennas in our bistatic experiment are active L-Band patch antennas with quasi-hemispherical radiation patterns (see Appendix C), and although a single section of their nominal gain pattern in elevation has been appended, a detailed description of $G_{\theta,\phi}$ over a whole hemisphere was not available.

In what follows, we will try to compensate for this lack of knowledge and make inferences about the antenna patterns and their relative orientation relying solely on our measurements. We know that both antennas have identical radiation patterns and that they are aligned in the vertical with oppositely oriented polar axes (not that they are necessarily aligned in azimuth). Our strategy consists in using direct signals collected from a variety of angles to probe the shape of the antenna radiation pattern [see expression (3.3.1) in Chapter I, assuming $P_tG_t/R_0^2$ a constant]. The observed antenna gain shape will be fit to a simple model that prescribes a “dipole-like” gain decay for large incidence angles combined with an ellipsoidal dependence in azimuth, i.e.:

$$G(\theta, \phi) = \frac{\cos^k(\theta)}{\sqrt{1 - e^2 \cos^2(\phi - \phi_0)}}$$  \hspace{1cm} (B.1)

The antenna gain model parameters $R$ (roll-off), $e$ (eccentricity) and $\phi_0$ are to be determined from observations. We will then use reflected signals and the gain model in (B.1) to determine the relative azimuth between the up and down-looking antenna frames.

\footnote{While the maximum difference in power output $P_t$ between GPS Block II/IIA satellites lies about 1 dB, signals levels received on Earth will typically vary another 1.5 dB due to slant range and transmitting antenna beam shaping [Edgar, 1998].}
Determination of antenna gain model parameters

Measurements of the direct signal power collected during the bistatic experiment are shown in Figure B.1 as a function of incidence angle for all satellites and azimuth angles. Superposed in black is a “dipole-like” curve proportional to $\cos^p(\theta)$. Figure B.2 shows the direct signal power levels as a function of azimuth for all satellites and elevations angles. Measurements have been grouped into three elevation bands (from 70 to 50, from 50 to 35, and from 35 to 20 degrees), and color-coded accordingly.

Figure B.1 – Zenithal antenna pattern (dB, uplooking)

Figure B.2 – Azimuthal antenna pattern (dB, uplooking)
We use a non-linear fit procedure to determine the roll-off exponent $R$, the azimuthal eccentricity $e$ and the direction of azimuthal maximum gain $\phi_{0}^{up}$ in (B.1) from power measurements collected by the up-looking antenna, yielding the results shown on Table B.1.

Table B.1 – Antenna gain model fit parameters (up-looking)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>1.0</td>
</tr>
<tr>
<td>$e$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\phi_{0}^{up}$</td>
<td>1.50 rad</td>
</tr>
</tbody>
</table>

Figure B.3 above illustrates the results for antenna pattern corrections. The upper plot shows GPS direct power measurements. The middle plot shows the model pattern corrections, derived from (B.1) using parameters in Table B.1. The lower plot shows GPS power measurements after the model pattern correction. The dispersion in beam pattern corrected GPS power in Fig. B.3 is attributed to mismodeling errors and intersatellite signal variability (i.e. the $P_{G}/R_{0}^{2}$ factor, which we assumed constant). Mismodeling errors get worse at lower elevations, suggesting the presence of stronger multipath as signals travel closer to the platform fuselage. From what precedes, we conclude that a
simple antenna gain model like (B.1) can be used to correct for major observed power variations due to changes in receiving antenna directivity with azimuth and elevation, but will not be able to correct for multipath effects, which overall leave an uncertainty of 0.8 dB (1-σ) in power measurements.

Determination of up/down antenna relative azimuth

To determine $\phi_0^{\text{down}}$, the angle of azimuthal maximum gain that the downlooking antenna axes form with the aircraft frame, we select a flight segment over a smooth and uniform surface (i.e. St Lawrence flight) and fit scattered peak power measurements to a gain model such as (B.1) fed with $R, e$ parameters from Table B.1.

The dispersion of power measurements about the antenna gain model is naturally higher for scattered signals (see Figure B.4, green curve to inner ring is model fit to reflected
power measurements), but a non-linear least squares procedure nevertheless succeeds in
providing an estimate for $\phi_{0}^{\text{down}}$ (see Table B.2).

Table B.2 – Antenna gain model fit parameters (down-looking)

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
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<td>R</td>
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</tr>
<tr>
<td>e</td>
<td>0.8</td>
</tr>
<tr>
<td>$\phi_{0}^{\text{down}}$</td>
<td>0.50 rad</td>
</tr>
</tbody>
</table>

Antenna pattern correction

The required antenna pattern correction factor $G_{r}^{\text{up}}/G_{r}^{\text{down}}$ required for the construction of scattered GPS waveforms from direct and reflected correlation power measurements is:

$$
\frac{G_{r}^{\text{up}}}{G_{r}^{\text{down}}} = \frac{\cos(\theta_{\text{up}})}{\cos(\theta_{\text{down}})} \left( \frac{1-e^2 \cos^2 (\phi - \phi_{0}^{\text{down}})}{1-e^2 \cos^2 (\phi - \phi_{0}^{\text{up}})} \right)^{1/2}
$$

(B.2)

Where $\theta_{\text{up}}, \theta_{\text{down}}$ refer to the direct and reflected wave incidence angles relative to the up and down-looking antenna body axes (the azimuth angle $\phi$ is the same for both waves in both frames, see Appendix D).
Appendix C – Antenna radiation pattern

The figure below shows an elevation cut in the nominal gain pattern for the bistatic experiment antenna [Sensor Systems S67-1575-(1)39] for rotating linearly polarized radiation. The outer envelope in this diagram gives the antenna nominal gain for linearly polarized radiation. The depth between the inner and outer envelopes gives the antenna axial ratio –or the quality of its circular polarization. The nomograph on the right of Figure C.1 is used to convert graph readings of linear polarization gain into circular polarization gain via axial ratios.

Figure C.1 – GPS antenna pattern for a rotating LP source (zenithal cut, from Sensor Systems Inc.)
<table>
<thead>
<tr>
<th>Elevation (degrees)</th>
<th>Lin. Gain (dB)</th>
<th>Axial Ratio (dB)</th>
<th>Correction (dB)</th>
<th>Cir. Gain (dB)</th>
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Appendix D – Earth fixed and body fixed frames

Transformation of LOS (line-of-sight) vectors between earth fixed (NED) and body fixed (BF) frames

Transforming line-of-sight unit vectors between coordinate frames amounts to building the transformation matrix that links both systems via, for example, the multiplication of ordered rotations about certain axes (see Figure D.1 for definitions of axes and angles in earth fixed –left- and body-fixed –right- coordinate frames). For our problem, we will use the aircraft attitude angles to specify the orientation of the antenna body-fixed frame relative to the local tangent NED frame using the following convention for angles:

i) heading, the aircraft bearing in the local NED frame, clockwise with respect to North or right handed about Z’,

ii) pitch, the aircraft pitch angle, right handed about the right wing axis,

iii) roll, the aircraft roll angle, right handed about the nose axis.

Figure D.1 – Definition. local topographic North-East-Down (NED) and Body-Fixed (BF) frames
Thus, aircraft attitude information (head, pitch and roll from an inertial system) will be combined with knowledge of the positions of the transmitting and receiving antennas (satellite elevation and azimuth angles in local frame), to obtain the directions of arrival of GPS signals (elevation and azimuth) relative to the antenna axes. The transformation is effected as:

\[
\begin{align*}
Q_z(r) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(r) & \sin(r) \\ 0 & -\sin(r) & \cos(r) \end{bmatrix} \\
Q_y(p) &= \begin{bmatrix} \cos(p) & 0 & -\sin(p) \\ 0 & 1 & 0 \\ \sin(p) & 0 & \cos(p) \end{bmatrix} \\
Q_z(h) &= \begin{bmatrix} \cos(h) & \sin(h) & 0 \\ -\sin(h) & \cos(h) & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{align*}
\]

\[
R = Q_z(r)Q_y(p)Q_z(h) R'
\]
Appendix E – Scattering and Fourier amplitudes

On the relation between scattering amplitudes $\bar{F}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}})$ and Fourier amplitudes

$$\bar{E}(\vec{k}_\perp).$$

$$\bar{E}(\vec{r}, 0) = \bar{F}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) \frac{e^{ik_r}}{r}$$

$$\bar{E}(\vec{r}, 0) = \left\{ \begin{array}{l} \bar{E}(\vec{k}_\perp) e^{ik_{\perp}} d^2k_{\perp} \\ \text{Both evaluated on } z=0 \text{ plane} \end{array} \right.$$  

$$\frac{e^{ik_r}}{r} = \frac{i}{2\pi} \int_{k_\perp} e^{ik_{\perp}} d^2k_{\perp}$$  

Fourier expansion of a spherical wave [Brekhovskikh, 1980]

$$\bar{E}(\vec{k}_\perp) = \frac{1}{(2\pi)^2} \int_{z=0} d^2r \bar{E}(\vec{r}, 0) e^{-ik_{\perp}z} =$$

$$= \frac{1}{(2\pi)^2} \int_{z=0} d^2r \bar{F}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) \frac{e^{ik_r}}{r} e^{-ik_{\perp}z}$$

$$= \frac{1}{(2\pi)^2} \int_{z=0} d^2r \int_{z=0} \bar{E}(\vec{k}_{\parallel}') e^{ik_{\parallel}z} e^{-ik_{\perp}z} d^2k_{\parallel}'$$

$$\frac{1}{(2\pi)^2} \int_{z=0} d^2r \bar{F}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) \frac{i}{2\pi} \int_{k_\perp} e^{i(k_{\perp} - k_{\parallel})z} d^2k_{\perp} = \frac{1}{(2\pi)^2} \int_{z=0} d^2r \int_{z=0} \bar{E}(\vec{k}_{\parallel}') e^{i(k_{\parallel} - k_{\perp})z} d^2k_{\parallel}'$$

$$\bar{F}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) \frac{i}{2\pi k_\perp} \int_{z=0} \frac{1}{k_\perp} \delta(k_{\perp} - k_{\parallel}) d^2k_{\perp} = \int_{z=0} \bar{E}(\vec{k}_{\parallel}) \delta(\vec{k}_{\perp} - \vec{k}_{\parallel}) d^2k_{\perp}$$

$$\bar{F}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}) \frac{i}{2\pi k_\perp} = \bar{E}(\vec{k}_\perp)$$  

(E.1)
Appendix F – Reference framework

Reference framework:

The unit vectors that describe the propagation and polarization state of a traveling electromagnetic disturbance (all angles relative to the global –mean surface- frame):

\[
\vec{n}_{\text{inc}} = \begin{bmatrix} 0 \\ \sin \theta \\ -\cos \theta \end{bmatrix} \quad \vec{h} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 0 \\ \cos \theta \\ \sin \theta \end{bmatrix}
\]

\[
\vec{n}_{\text{scat}} = \begin{bmatrix} \sin \theta_s \cos \phi_s \\ \sin \theta_s \sin \phi_s \\ \cos \theta_s \end{bmatrix} \quad \vec{h}_s = \begin{bmatrix} \sin \phi_s \\ -\cos \phi_s \\ 0 \end{bmatrix} \quad \vec{v}_s = \begin{bmatrix} -\cos \theta_s \cos \phi_s \\ -\cos \theta_s \sin \phi_s \\ \sin \theta_s \end{bmatrix}
\]

\hspace{1cm} (F.1)
Appendix G – Geometric pre-factors in KA

Calculation of geometric pre-factors $C_{sp}$ in the Kirchhoff approximation, taking into account the relation between global (mean surface) and local (tilted facet) frame angles.

Figure G.1 - Surface frame convention

Incident field can be decomposed in the surface frame into $H,V$ components as:

$$\vec{E} = \vec{E}_H + \vec{E}_V$$  \hspace{1cm} (G.1)

Where:

$$\vec{E}_H = E_H \frac{\vec{n}_{inc} \times \hat{z}}{|\vec{n}_{inc} \times \hat{z}|}$$

$$\vec{E}_V = E_V \frac{\vec{n}_{inc} \times \hat{z}}{|\vec{n}_{inc} \times \hat{z}|} \times \vec{n}_{inc}$$  \hspace{1cm} (G.2)

Each of these can be projected onto the local (facet) frame and be resolved into components perpendicular and parallel to the facet. Define:

$$\hat{\chi} = \frac{\vec{n}_{inc} \times \hat{z}'}{|\vec{n}_{inc} \times \hat{z}'|}$$  \hspace{1cm} (G.3)

Then the local horizontal and vertical electric field components:
\[ E_h = \hat{x}' \cdot (\vec{E}_H + \vec{E}_v) = \hat{x}' \cdot \hat{p}_0 \]
\[ E_v = \hat{x}' \times \vec{n}_{inc} \cdot (\vec{E}_H + \vec{E}_v) = \hat{x}' \times \vec{n}_{inc} \cdot \hat{p}_0 \]

(G.4)

And thus the stationary phase coefficient, evaluated at the local specular point, from (5.1.4) with \( \vec{n}_{sp} \) the local facet normal:

\[
\hat{C}_{sp} = \hat{u}_k \times \left[ \mathcal{R}_k E_h \left( \hat{u}_k \times (\vec{n}_{sp} \times (\vec{n}_{ref} \times \hat{x})) - (\vec{n}_{sp} \times \hat{x}) \right) \right] + \mathcal{R}_v E_v \left( \hat{u}_k \times (\vec{n}_{sp} \times \hat{x}) + \vec{n}_{sp} \times (\vec{n}_{ref} \times \hat{x}) \right)
\]

(G.5)

Make the substitutions \( x \rightarrow x' \), \( \vec{n}_{sp} \rightarrow z' \)

\[
\hat{C}_{sp} = \hat{u}_k \times \left[ \mathcal{R}_k \hat{x}' \cdot \hat{p}_0 \left( \hat{u}_k \times (\vec{z}' \times (\vec{n}_{ref} \times \hat{x}')) - (\vec{z}' \times \hat{x}') \right) \right] + \mathcal{R}_v \hat{x}' \times \vec{n}_{inc} \cdot \hat{p}_0 \left( \hat{u}_k \times (\vec{z}' \times \hat{x}') + \vec{z}' \times (\vec{n}_{ref} \times \hat{x}') \right)
\]

(G.6)

And with a little algebra (using Figure G.1 above)

\[
= \left[ \mathcal{R}_k \hat{x}' \cdot \hat{p}_0 \left( \hat{u}_k \times (\vec{z}' \times (\vec{n}_{ref} \times \hat{x}')) - \hat{u}_k \times (\vec{z}' \times \hat{x}') \right) \right] + \mathcal{R}_v \hat{x}' \times \vec{n}_{inc} \cdot \hat{p}_0 \left( \hat{u}_k \times (\vec{z}' \times \hat{x}') + \hat{z}' \times (\vec{n}_{ref} \times \hat{x}') \right)
\]

i) \( \hat{u}_k \times \left( \hat{u}_k \times (\vec{z}' \times (\vec{n}_{ref} \times \hat{x}')) \right) = -\vec{z}' \cdot \vec{n}_{ref} \hat{u}_k \times (\hat{u}_k \times \hat{x}') = \vec{n}_{ref} \cdot \vec{z}' \hat{x}' \)

ii) \( \hat{u}_k \times (\vec{z}' \times \hat{x}') = -\hat{u}_k \cdot \vec{z}' \hat{x}' \)

iii) \( \hat{u}_k \times \left( \hat{u}_k \times (\vec{z}' \times \hat{x}') \right) = -\hat{u}_k \cdot \vec{z}' \hat{u}_k \times \hat{x}' \)

iv) \( \hat{u}_k \times \vec{z}' \times (\vec{n}_{ref} \times \hat{x}') = -\vec{n}_{ref} \cdot \vec{z}' \hat{u}_k \times \hat{x}' \)

(G.7)

Introduce an output polarization vector \( p \) (and the proviso that \( \vec{u}_k = \vec{n}_{ref} \)):

\[ C_{ph} = \hat{p} \cdot \hat{C}_{sp} = 2\vec{n}_{ref} \cdot \vec{z}' \left[ \mathcal{R}_k \left( \hat{x}' \cdot \hat{p}_0 \right) \left( \hat{p} \cdot \hat{x}' \right) - \mathcal{R}_v \left( \hat{x}' \times \vec{n}_{inc} \cdot \hat{p}_0 \right) \left( \hat{p} \cdot \vec{n}_{ref} \times \hat{x}' \right) \right] \]

(G.8)

And the following identities (substitutions and \( \vec{A} \times \vec{B} \cdot \vec{C} = \vec{A} \cdot (\vec{B} \times \vec{C}) \)):

\[
\hat{x}' = \frac{\vec{n}_{inc} \times \vec{z}'}{|\vec{n}_{inc} \times \vec{z}'|} = \frac{\vec{n}_{inc} \times \vec{n}_{ref}}{|\vec{n}_{inc} \times \vec{n}_{ref}|} \quad \text{since} \quad \vec{z}' = \frac{k}{q} (\vec{n}_{ref} - \vec{n}_{inc})
\]

(G.9)

\[ \hat{x}' \cdot \hat{p}_0 = \frac{\vec{n}_{inc} \times \vec{n}_{ref}}{|\vec{n}_{inc} \times \vec{n}_{ref}|} \times \vec{n}_{inc} \cdot \hat{v} = \frac{\vec{n}_{ref} \cdot \hat{v}}{|\vec{n}_{inc} \times \vec{n}_{ref}|} \]

for H-pol incidence, \( p_0 = x \).
\[ \hat{x} \times \vec{n}_{inc} \cdot \hat{p}_0 = \left( \vec{n}_{inc} \times \vec{n}_{ref} \right) \hat{h} \times \vec{n}_{inc} = \left( \vec{n}_{ref} \cdot \hat{h} \right) \]

\[ \hat{x} \cdot \vec{p}_0 = \left( \vec{n}_{inc} \times \vec{n}_{ref} \right) \hat{h} \times \vec{n}_{inc} = -\left( \vec{n}_{ref} \cdot \hat{h} \right) \]

for V-pol incidence, \( p_0 = \vec{v} \)

\[ \hat{x} \times \vec{n}_{inc} \cdot \vec{p}_0 = \left( \vec{n}_{inc} \times \vec{n}_{ref} \right) \hat{v} \times \vec{n}_{inc} = \left( \vec{n}_{ref} \cdot \hat{v} \right) \]

for H-pol output, \( p = \vec{x} \)

\[ \hat{p} \cdot \hat{x}' = \vec{n}_{ref} \times \hat{v}_s \left( \vec{n}_{inc} \times \vec{n}_{ref} \right) \hat{h} \times \vec{n}_{inc} = \left( \vec{n}_{inc} \cdot \hat{h} \right) \]

\[ \hat{p} \cdot \vec{n}_{ref} \times \vec{x}' = \vec{x}' \cdot \hat{p} \times \vec{n}_{ref} = \left( \vec{n}_{inc} \times \vec{n}_{ref} \right) \hat{h} \times \vec{n}_{inc} = \left( \vec{n}_{inc} \cdot \hat{h} \right) \]

\[ \hat{p} \cdot \hat{x}' = \vec{h}_s \times \vec{n}_{inc} \left( \vec{n}_{inc} \times \vec{n}_{ref} \right) \]

for H-pol output, \( p = \vec{v} \)

\[ \hat{p} \cdot \vec{n}_{ref} \times \vec{x}' = \vec{x}' \cdot \hat{p} \times \vec{n}_{ref} = \left( \vec{n}_{inc} \times \vec{n}_{ref} \right) \hat{v}_s \times \vec{n}_{ref} = \left( \vec{n}_{inc} \cdot \hat{v}_s \right) \]

\[ \hat{p} \cdot \vec{n}_{ref} \times \vec{x}' = \vec{x}' \cdot \hat{p} \times \vec{n}_{ref} = \left( \vec{n}_{inc} \times \vec{n}_{ref} \right) \hat{v}_s \times \vec{n}_{ref} = \left( \vec{n}_{inc} \cdot \hat{v}_s \right) \] (G.10)

To obtain [Wu & Fung, 1972]:

\[ C_{hh} = -\frac{2 \cos \beta}{\sin^2 2\beta} \left[ \Re_h \left( \vec{n}_{ref} \cdot \hat{v} \right) \left( \vec{n}_{inc} \cdot \hat{v}_s \right) + \Re_v \left( \vec{n}_{ref} \cdot \hat{h} \right) \left( \vec{n}_{inc} \cdot \hat{h}_s \right) \right] \]

\[ C_{hv} = \frac{2 \cos \beta}{\sin^2 2\beta} \left[ \Re_h \left( \vec{n}_{ref} \cdot \hat{v} \right) \left( \vec{n}_{inc} \cdot \hat{h}_s \right) - \Re_v \left( \vec{n}_{ref} \cdot \hat{h} \right) \left( \vec{n}_{inc} \cdot \hat{v}_s \right) \right] \]

\[ C_{vh} = \frac{2 \cos \beta}{\sin^2 2\beta} \left[ \Re_h \left( \vec{n}_{ref} \cdot \hat{h} \right) \left( \vec{n}_{inc} \cdot \hat{v}_s \right) - \Re_v \left( \vec{n}_{ref} \cdot \hat{v} \right) \left( \vec{n}_{inc} \cdot \hat{h}_s \right) \right] \]

\[ C_{vv} = -\frac{2 \cos \beta}{\sin^2 2\beta} \left[ \Re_h \left( \vec{n}_{ref} \cdot \hat{h} \right) \left( \vec{n}_{inc} \cdot \hat{h}_s \right) + \Re_v \left( \vec{n}_{ref} \cdot \hat{v} \right) \left( \vec{n}_{inc} \cdot \hat{v}_s \right) \right] \] (G.11)

Where the dot products in the specular point coefficients are given by (see Appendix F):

\( \left( \vec{n}_{inc} \cdot \hat{v}_s \right) = -\sin \theta \cos \phi \sin \phi_s - \cos \theta \sin \phi \)

\( \left( \vec{n}_{ref} \cdot \hat{v} \right) = \cos \theta \sin \phi \sin \phi_s + \sin \theta \cos \phi \)
\[
\left( \hat{n}_{\text{inc}} \cdot \hat{h} \right) = -\sin \theta \cos \phi_s
\]
\[
\left( \hat{n}_{\text{ref}} \cdot \hat{h} \right) = \sin \theta \cos \phi_s
\]  
\hfill (G.12)

And the Fresnel reflection coefficients are evaluated at the local reflection angle \( \beta \), such that:
\[
|\hat{n}_{\text{inc}} \times \hat{n}_{\text{ref}}| = \sin 2\beta
\]  
\hfill (G.13)

\[
\hat{n}_{\text{ref}} \cdot \hat{\varphi}' = \cos \beta = \frac{q}{2k} = \sqrt{(1 - \sin \theta \sin \theta_s \sin \phi_s \cos \theta \cos \theta_s) / 2}
\]  
\hfill (G.14)

In practice, the calculation of coefficients (G.11) proves to be numerically unstable. Further simplification of the tangent plane pre-factors \( C_{SP} \) is possible if one takes as extra assumption that surface slopes are small. In this case, the local (facet) and mean surface frames become indistinguishable and
\[
\hat{n}_{\text{inc}} \cdot \hat{h} = \hat{n}_{\text{ref}} \cdot \hat{h} = 0
\]
\[
\hat{n}_{\text{inc}} \cdot \hat{\varphi} = \cos(\pi / 2 + 2\beta) = -\sin 2\beta
\]
\[
\hat{n}_{\text{ref}} \cdot \hat{\varphi} = \cos(\pi / 2 - 2\beta) = \sin 2\beta
\]  
\hfill (G.15)

Leading to:
\[
C_{hh} = 2 \cos \beta \mathcal{R}_h = \frac{q}{k} \mathcal{R}_h \quad C_{hv} = 0
\]
\[
C_{hh} = 2 \cos \beta \mathcal{R}_v = \frac{q}{k} \mathcal{R}_v \quad C_{hv} = 0 \quad \text{(i.e. scalar Kirchhoff approximation)}
\]  
\hfill (G.16)

In the scalar Kirchhoff approximation, slope terms in the local coordinate vectors are neglected and thus polarization changes at local facets ignored. In this case, the polarized scattered fields computed have the same form as those obtained using the scalar Helmholtz integral.
Appendix H – Geometric pre-factors in SPM

Matching the tangential electric $E$ and magnetic $H$ fields across a discontinuity: perturbation theory

The total (incident + reflected + scattered) vector fields $E_{T1}$ and $H_{T1}$ in the upper half-space (vacuum/air) and $E_{T2}$ and $H_{T2}$ in the lower half space (with permittivity $\varepsilon$) must satisfy the corresponding homogeneous wave equation in each half-space and the electromagnetic boundary conditions at the interface $z = \xi(x,y)$

$$\hat{n} \times (\vec{E}_{T2} - \vec{E}_{T1})|_{z=\xi(x,y)} = 0$$
$$\hat{n} \times (\vec{H}_{T2} - \vec{H}_{T1})|_{z=\xi(x,y)} = 0$$

(H.1)

Where $n$ is a unit vector normal to the surface (i.e. the local normal) rewritten in terms of the gradient of surface elevations $\gamma = \nabla_\perp \xi(x,y)$ as:

$$\hat{n} = \frac{\hat{e}_z - \nabla_\perp \xi}{\sqrt{1 + |\nabla_\perp \xi|^2}} \approx \hat{e}_z - \hat{\gamma}$$

(H.2)

![Figure H.1 – Surface local normal n](image)

We will represent the total fields as the sum of an unperturbed (specular) field $E_0$, which corresponds to the solution of Fresnel reflection (refraction) at a plane interface, plus a
field perturbation $E$ that represents the rough surface correction to first order in heights $\xi$ and slopes $\gamma$

\[\begin{align*}
\vec{E}_{T1} &= \vec{E}_{01} + \vec{E}_1 \\
\vec{E}_{T2} &= \vec{E}_{02} + \vec{E}_2 \\
\vec{H}_{T1} &= \vec{H}_{01} + \vec{H}_1 \\
\vec{H}_{T2} &= \vec{H}_{02} + \vec{H}_2
\end{align*}\] (H.3)

The unperturbed fields are built to satisfy the boundary conditions on the plane $z=0$:

\[\begin{align*}
\hat{e}_z \times (\vec{E}_{20} - \vec{E}_{10})|_{z=0} &= 0 \\
\hat{e}_z \times (\vec{H}_{20} - \vec{H}_{10})|_{z=0} &= 0
\end{align*}\] (H.4)

Which provide the well known Fresnel amplitude and Snell-law\(^8\) relationships. Given an incident monochromatic plane wave traveling along a direction specified by $\mathbf{n}_{\text{inc}}$ on the $x=0$ plane with polarization vector $\mathbf{p}_0$ (i.e. take $\mathbf{p}_0 = \mathbf{e}_x$ axis for the TM and TE fields), the unperturbed fields are constructed using Maxwell relations for plane waves (see Section 4, MKS units):

\[\begin{align*}
\vec{H}_{01} &= \vec{p}_0 (e^{i\kappa_{\text{inc}} r} + \mathcal{R}_{e} e^{i\kappa_{\text{ref}} r}) / Z_0 \\
\vec{H}_{02} &= \vec{p}_0 (1 + \mathcal{R}_{e}) e^{i\kappa_{\text{max}} r} / Z_0 \\
\vec{E}_{01} &= \quad [ (\vec{n}_{\text{inc}} \times \vec{p}_0) e^{i\kappa_{\text{inc}} r} + \mathcal{R}_{\ell} (\vec{n}_{\text{ref}} \times \vec{p}_0) e^{i\kappa_{\text{ref}} r} ] \\
\vec{E}_{02} &= -e^{-i/2} (1 + \mathcal{R}_{\ell}) (\vec{n}_{\text{trans}} \times \vec{p}_0) e^{i\kappa_{\text{trans}} r}
\end{align*}\] (H.5a)

\[\begin{align*}
\vec{H}_{01} &= \vec{p}_0 (e^{i\kappa_{\text{inc}} r} + \mathcal{R}_{e} e^{i\kappa_{\text{ref}} r}) \\
\vec{H}_{02} &= \vec{p}_0 (1 + \mathcal{R}_{e}) e^{i\kappa_{\text{max}} r} \\
\vec{E}_{01} &= \quad [ (\vec{n}_{\text{inc}} \times \vec{p}_0) e^{i\kappa_{\text{inc}} r} + \mathcal{R}_{\ell} (\vec{n}_{\text{ref}} \times \vec{p}_0) e^{i\kappa_{\text{ref}} r} ] / Z_0 \\
\vec{E}_{02} &= e^{-i/2} (1 + \mathcal{R}_{\ell}) (\vec{n}_{\text{trans}} \times \vec{p}_0) e^{i\kappa_{\text{trans}} r} / Z_0
\end{align*}\] (H.5b)

Where $\mathcal{R}$ are the Fresnel reflection coefficients. Assuming that surface heights are small, we can expand the total fields at the boundary $\xi(x,y)$ in a Taylor series about $z=0$

\[\begin{align*}
\vec{E}_T|_{z=\xi(x,y)} &= \vec{E}_T|_{z=0} + \xi \frac{\partial \vec{E}_T}{\partial z} \bigg|_{z=0} \\
\vec{H}_T|_{z=\xi(x,y)} &= \vec{H}_T|_{z=0} + \xi \frac{\partial \vec{H}_T}{\partial z} \bigg|_{z=0}
\end{align*}\] (H.6)

And enforce (H.1) up to first order in surface heights, slopes and field perturbations, to obtain:

\[^{8}\text{Snell’s Law is } \sqrt{\varepsilon} \sin \theta’ = \sin \theta\]
Where a new notation has been introduced for field discontinuities at \( z=0 \):

\[
\Delta \vec{E} = \vec{E}_2 - \vec{E}_1 \quad \Delta \vec{E}_0 = \vec{E}_{02} - \vec{E}_{01} \\
\Delta \vec{H} = \vec{H}_2 - \vec{H}_1 \quad \Delta \vec{H}_0 = \vec{H}_{02} - \vec{H}_{01}
\]  

The approximate boundary conditions (H.7) for the scattered fields discontinuities \( \Delta \vec{E} \) and \( \Delta \vec{H} \) on the \( z=0 \) plane are given in terms of known unperturbed field differences \( \Delta \vec{E}_0 \) and \( \Delta \vec{H}_0 \) (see Appendix H1), surface heights \( \xi \) and slopes \( \gamma \). In translating these conditions onto a Fourier domain (see Appendix H2), the differential relation between surface heights and slopes becomes an algebraic one and the vector boundary conditions simplify to:

**TM (v-pol):**

\[
\hat{e}_z \times \left( \vec{E}_2(\vec{k}) - \vec{E}_1(\vec{k}) \right) \bigg|_{z=0} = \vec{J}_E^v \\
\hat{e}_z \times \left( \vec{H}_2(\vec{k}) - \vec{H}_1(\vec{k}) \right) \bigg|_{z=0} = \vec{J}_M^v / Z_0
\]

**TE (h-pol):**

\[
\hat{e}_z \times \left( \vec{E}_2(\vec{k}) - \vec{E}_1(\vec{k}) \right) \bigg|_{z=0} = \vec{J}_E^h \\
\hat{e}_z \times \left( \vec{H}_2(\vec{k}) - \vec{H}_1(\vec{k}) \right) \bigg|_{z=0} = \vec{J}_M^h / Z_0
\]

The solution for the scattered field Fourier amplitudes \( \vec{E}_1(\vec{k}), \vec{E}_2(\vec{k}), \vec{H}_1(\vec{k}), \vec{H}_2(\vec{k}) \) in the upper and lower media in (H.9) are obtained in a straightforward way when we consider scattering into vertical (TM) and horizontal (TE) polarizations separately. Introduce the output polarization unit vector \( \hat{p} \), orthogonal to the propagation vectors \( \vec{n}_{\text{scat}}, \vec{n}_{\text{trans}} \) as well as to the normal unit vector \( \vec{e}_z \) (all of which are in the plane of scattering, see Figure H.2 below):

\[
\hat{p} = \begin{bmatrix} \sin \phi_s \\ -\cos \phi_s \\ 0 \end{bmatrix} \quad \hat{n}_{\text{scat}} = \frac{1}{k} \{ \vec{k}, \vec{\kappa}_z \} = \begin{bmatrix} \sin \theta_s \cos \phi_s \\ \sin \theta_s \sin \phi_s \\ \cos \theta_s \end{bmatrix}
\]
Where $\phi_s$ is the azimuthal angle of scattering and $\theta_s$ the polar scattering angle.

![Figure H.2 – Geometry of scattering](image)

**i) VV scatter (TM into TM)**

Solve for the (transversal) magnetic Fourier amplitude $\mathbf{H}_1(\kappa)$ using Maxwell relations:

\[
\vec{E}_1(\vec{k}) = -\vec{n}_{\text{scat}} \times \vec{H}_1(\vec{k}) Z_0 \\
\vec{E}_2(\vec{k}) = -\epsilon^{-1/2} \vec{n}_{\text{trans}} \times \vec{H}_2(\vec{k}) Z_0
\]

(H.11)

And find, in substituting (H.11) into (H.9a), that:

\[
\vec{p} \cdot \vec{H}_1(\vec{k}) = \frac{\kappa_z' \vec{p} \cdot (\hat{e}_z \times J'_M) - \epsilon k \vec{p} \cdot J'_E}{(\kappa_z \epsilon + \kappa_{z'}')} 1/Z_0
\]

(H.12)

The scattered field Fourier amplitude into the TM component in the upper half space is equal to the projection of $\mathbf{H}_1(\kappa)$ along the unit polarization vector $\vec{p}$. Insert the appropriate surface currents (Appendix H2) and obtain the copolarized TM fields scattered into the upper medium:\(^9\)

\[
\text{Use } \mathcal{R}_s(\theta_s) = (\kappa_z - \kappa_{z'})'(\kappa_z + \kappa_{z'}), \text{ with the convention } \kappa_z = |\kappa_z|, \kappa_{z'} = \lvert \kappa_{z'}'\rvert.
\]

9
\[ \hat{p} \cdot \vec{H}_i(\vec{k}) = -i \xi (\vec{\theta}_\perp) \frac{\varepsilon - 1}{2\varepsilon Z_0} \left[ (1 - \mathcal{R}_v(\theta))(1 - \mathcal{R}_s(\theta))\varepsilon_j k_z \sin \phi_j - (1 + \mathcal{R}_v(\theta))(1 + \mathcal{R}_s(\theta))\frac{k_\kappa}{\kappa_\perp} \right] \]

**ii) HV scatter (TM into TE)**

Solve for the (transversal) electric Fourier amplitude \( \vec{E}_i(\kappa) \) using Maxwell relations:

\[
\vec{H}_i(\kappa) = \vec{n}_{\text{scat}} \times \vec{E}_i(\kappa) / Z_0 \\
\vec{H}_\perp(\kappa) = \varepsilon^{1/2} \vec{n}_{\text{trans}} \times \vec{E}_\perp(\kappa) / Z_0 \\
\text{(H.14)}
\]

And find, in substituting (H.14) into (H.9a), that:

\[
\vec{p} \cdot \vec{E}_i(\kappa) = \frac{\kappa_\perp}{\kappa_\parallel} \vec{p} \cdot (\vec{e}_z \times \vec{J}_E^\prime) + k \vec{p} \cdot \vec{J}_M
\]

\[
\text{(H.15)}
\]

Insert the appropriate surface currents (Appendix H2) and obtain the scattered field Fourier amplitudes into the cross-polarized TE component propagating along \( \vec{n}_{\text{scat}} \):

\[
\vec{p} \cdot \vec{E}_i(\kappa) = -i \xi (\vec{\theta}_\perp) \frac{\varepsilon - 1}{2\kappa_\perp} k \left(1 - \mathcal{R}_v(\theta))(1 - \mathcal{R}_s(\theta))\varepsilon \cos \phi_j \right)
\]

\[
\text{(H.16)}
\]

**iii) VH scatter (TE into TM)**

We can use the general solution (H.12) obtained above with the substitutions \( \vec{J}_M \to \vec{J}_M^\prime \) and \( \vec{J}_E \to 0 \). The scattered field Fourier amplitudes into TM waves are given by:

\[
\vec{p} \cdot \vec{H}_i(\kappa) = -i \xi (\vec{\theta}_\perp) \frac{\varepsilon - 1}{2Z_0} k \left(1 + \mathcal{R}_v(\theta))(1 - \mathcal{R}_s(\theta))\varepsilon \cos \phi_j \right)
\]

\[
\text{(H.17)}
\]

**iv) HH scatter (TE into TE)**

We can use the general solution (H.15) obtained above with the substitutions \( \vec{J}_M \to \vec{J}_M^\prime \) and \( \vec{J}_E \to 0 \). The scattered field Fourier amplitudes into TE waves are given by:
\[ \vec{p} \cdot \vec{E}_i(\vec{k}) = i \vec{\xi} (\vec{q}_\perp) \frac{\varepsilon - 1}{2\kappa_z} k^2 (1 + \Re_h(\theta_i))(1 + \Re_h(\theta_s)) \sin \phi_s \] (H.18)

In summary, depending on the output and input polarizations of the radiation interacting with the surface, the scattered field Fourier amplitudes in first order perturbation theory will be given by:

\[ \vec{E}_{\text{scat}, \text{pp}_b}(\vec{k}) = \begin{cases} 
\vec{p} \cdot \vec{H}_i(\vec{k}) Z_0 & \text{for VV from (H.13)} \\
\vec{p} \cdot \vec{E}_i(\vec{k}) & \text{for HV from (H.16)} \\
\vec{p} \cdot \vec{H}_i(\vec{k}) Z_0 & \text{for VH from (H.17)} \\
\vec{p} \cdot \vec{E}_i(\vec{k}) & \text{for HH from (H.18)} 
\end{cases} \]

Or

\[ \vec{E}_{\text{scat}, \text{pp}_b}(\vec{k}) = c_{\text{pp}_b}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}; \varepsilon) \vec{\xi}(\vec{q}_\perp) \] (H.19)

Where:

\[ c_{\text{pp}_b}(\vec{n}_{\text{inc}}, \vec{n}_{\text{scat}}; \varepsilon) = \frac{i k^2}{2\kappa_z} (\varepsilon - 1) f_{\text{pp}_b} \] (H.20)

\[ f_{\text{pp}_b} = \begin{cases} 
\frac{1}{\varepsilon} (1 + \Re_v(\theta_i))(1 + \Re_v(\theta_s)) \frac{k_1\kappa}{kk} - (1 - \Re_v(\theta_i))(1 - \Re_v(\theta_s)) \frac{k_2\kappa}{kk} \sin \phi_s & \text{VV} \\
-\frac{k}{k} (1 - \Re_v(\theta_i))(1 + \Re_h(\theta_s)) \cos \phi_s & \text{HV} \\
-\frac{\kappa}{k} (1 + \Re_h(\theta_i))(1 - \Re_v(\theta_s)) \cos \phi_s & \text{VH} \\
(1 + \Re_h(\theta_i))(1 + \Re_h(\theta_s)) \sin \phi_s & \text{HH} 
\end{cases} \]
Appendix H1 – Perturbed boundary conditions

Details on the calculation of the approximate boundary conditions (H.7)

If the incident wave is vertically/horizontally polarized, the zeroth-order field discontinuities \( \Delta E_0, \Delta H_0 \) and their normal derivatives in the right hand side of boundary conditions (H.7) can be found from (H.5a/b) as:

i) **TM (v-pol) incident field**

\[
\begin{align*}
\Delta E_0 \bigg|_{z=0} &= -\frac{k_j}{k\varepsilon} (\varepsilon - 1)(1 + \Re_\varepsilon) e^{\hat{d}_z} \hat{e}_z \\
\Delta H_0 \bigg|_{z=0} &= 0 \\
\frac{\partial \Delta E_0}{\partial z} \bigg|_{z=0} &= -i(\hat{e}_z \times \hat{p}_0) \frac{k_j}{k\varepsilon}^2 (\varepsilon - 1)(1 + \Re_\varepsilon) e^{\hat{d}_z} \hat{e}_z \\
\frac{\partial \Delta H_0}{\partial z} \bigg|_{z=0} &= -i\hat{p}_0 k_z (\varepsilon - 1)(1 - \Re_\varepsilon) e^{\hat{d}_z} / Z_0
\end{align*}
\]

(H1.1)$^{10}$

ii) **TE (h-pol) incident field**

\[
\begin{align*}
\Delta E_0 \bigg|_{z=0} &= 0 \\
\Delta H_0 \bigg|_{z=0} &= 0 \\
\frac{\partial \Delta E_0}{\partial z} \bigg|_{z=0} &= 0 \\
\frac{\partial \Delta H_0}{\partial z} \bigg|_{z=0} &= i\hat{e}_z k(\varepsilon - 1)(1 + \Re_\varepsilon) e^{\hat{d}_z} / Z_0
\end{align*}
\]

(H1.2)

Where the symbol \( \perp \) denotes the projection of the corresponding vector onto the \( z = 0 \) plane.

---

$^{10}$ To ease these derivations, use alternative expressions \( \Re_\varepsilon(\theta) = (\varepsilon k_z - k_z')/(\varepsilon k_z + k_z') \) and \( \Re_h(\theta) = (k_x - k_x')/(k_x + k_x') \) for the Fresnel v-pol and h-pol coefficients, with the convention \( k_z = |k_z|, \) \( k_z' = |k_z'|. \)
Appendix H2 – Electric and magnetic currents

Electromagnetic boundary conditions in Fourier domain: electric and magnetic currents $J_E, J_M$.

The approximate boundary conditions (H.7)

$$\hat{e}_z \times \Delta \tilde{E} \big|_{z=0} \approx \hat{y} \times \Delta \tilde{E}_0 \big|_{z=0} - \xi \hat{e}_z \times \frac{\partial \Delta \tilde{E}_0}{\partial z} \bigg|_{z=0}$$

$$\hat{e}_z \times \Delta \tilde{H} \big|_{z=0} \approx \hat{y} \times \Delta \tilde{H}_0 \big|_{z=0} - \xi \hat{e}_z \times \frac{\partial \Delta \tilde{H}_0}{\partial z} \bigg|_{z=0} \quad (H.2.1)$$

Are taken to the Fourier domain by means of the transformations (see Appendix H3)

$$\Delta \tilde{E} \big|_{z=0} = \tilde{E}_2(r',0) - \tilde{E}_1(r',0) = \iint_{z=0} \left( \tilde{E}_2(\tilde{k}) - \tilde{E}_1(\tilde{k}) \right) e^{i\tilde{ky}} d^2\tilde{k}$$

$$\Delta \tilde{H} \big|_{z=0} = \tilde{H}_2(r',0) - \tilde{H}_1(r',0) = \iint_{z=0} \left( \tilde{H}_2(\tilde{k}) - \tilde{H}_1(\tilde{k}) \right) e^{i\tilde{ky}} d^2\tilde{k} \quad (H.2.2)$$

A) for incident TM (V-pol) fields, using results from Appendix H1

$$\hat{e}_z \times \iint_{z=0} \left( \tilde{E}_2(\tilde{k}') - \tilde{E}_1(\tilde{k}') \right) e^{i\tilde{ky}} d^2\tilde{k}' = \frac{k}{kE} (e-1)(1+i\mathcal{R}) \left( \hat{e}_z \times \hat{y} + i\xi \hat{e}_z \times (\hat{e}_z \times \hat{p}_0)k_x \right) e^{i\tilde{ky}}$$

$$\hat{e}_z \times \iint_{z=0} d^2r e^{-i\tilde{ky}} \iint_{z=0} \left( \tilde{E}_2(\tilde{k}') - \tilde{E}_1(\tilde{k}') \right) e^{i\tilde{ky}} d^2\tilde{k}' =$$

$$= \frac{k}{kE} (e-1)(1+i\mathcal{R}) \iint_{z=0} d^2r e^{i\tilde{k}_0 \cdot \tilde{r}} \left( \hat{e}_z \times \hat{y} + i\xi \hat{e}_z \times (\hat{e}_z \times \hat{p}_0)k_x \right)$$

$$\hat{e}_z \times (2\pi)^2 \left( \tilde{E}_2(\tilde{k}) - \tilde{E}_1(\tilde{k}) \right) =$$

$$\frac{k}{kE} (e-1)(1+i\mathcal{R}) \left[ \hat{e}_z \times \iint_{z=0} d^2r e^{i\tilde{k}_0 \cdot \tilde{r}} \nabla \tilde{\xi}(\tilde{r}) + i\xi \hat{e}_z \times (\hat{e}_z \times \hat{p}_0)k_x \right]$$

$$\hat{e}_z \times (\tilde{E}_2(\tilde{k}) - \tilde{E}_1(\tilde{k})) = \frac{k}{kE} (e-1)(1+i\mathcal{R}) \left[ -\hat{e}_z \times (\tilde{k}_1 - \tilde{k}) + \hat{e}_z \times (\hat{e}_z \times \hat{p}_0)k_x \right] \tilde{\xi}(\tilde{k} - \tilde{k}_1)$$

$$\hat{e}_z \times (\tilde{E}_2(\tilde{k}) - \tilde{E}_1(\tilde{k})) = i\xi \tilde{\xi}(\tilde{k}_1) \frac{k}{kE} (e-1)(1+i\mathcal{R}) (\hat{e}_z \times \tilde{k}) \equiv \tilde{J}_E$$

(H2.3)
Analogously,

\[ \hat{e}_z \times \iint \left( \mathbf{H}_2(\mathbf{k'}) - \mathbf{H}_1(\mathbf{k'}) \right) e^{i\mathbf{k'} r} d^2 \mathbf{k'} = i\hat{e}_z \times \hat{p}_0 k_z (\mathbf{e} - 1)(1 - \mathbb{R}_t) e^{i\mathbf{h'} r} / Z_0 \]

\[ = i\hat{e}_z \times \hat{p}_0 k_z (\mathbf{e} - 1)(1 - \mathbb{R}_t) \iint d^2 r e^{i\mathbf{h'} r} \xi(\mathbf{r'}) e^{i\mathbf{h'} r} / Z_0 \]

\[ \hat{e}_z \times \left( \mathbf{H}_2(\mathbf{k}) - \mathbf{H}_1(\mathbf{k}) \right) = i\xi(\mathbf{q}_\perp) k_z (\mathbf{e} - 1)(1 - \mathbb{R}_t) (\hat{e}_z \times \hat{p}_0) / Z_0 = \tilde{J}_M / Z_0 \quad (H2.4) \]

B) for incident TE (H-pol) fields, using results from Appendix H1

\[ \hat{e}_z \times \iint \left( \mathbf{E}_2(\mathbf{k'}) - \mathbf{E}_1(\mathbf{k'}) \right) e^{i\mathbf{h'} r} d^2 \mathbf{k'} = 0 \]

\[ \hat{e}_z \times \left( \mathbf{E}_2(\mathbf{k}) - \mathbf{E}_1(\mathbf{k}) \right) = 0 \equiv \tilde{J}_E \quad (H2.5) \]

Analogously:

\[ \hat{e}_z \times \iint \left( \mathbf{H}_2(\mathbf{k'}) - \mathbf{H}_1(\mathbf{k'}) \right) e^{i\mathbf{h'} r} d^2 \mathbf{k'} = -i\xi(\mathbf{q}_\perp) k_z (\mathbf{e} - 1)(1 + \mathbb{R}_t) e^{i\mathbf{h'} r} / Z_0 \]

\[ = -i\hat{e}_z \times \hat{p}_0 k_z (\mathbf{e} - 1)(1 + \mathbb{R}_t) \iint d^2 r e^{i\mathbf{h'} r} \xi(\mathbf{r'}) e^{i\mathbf{h'} r} / Z_0 \]

\[ \hat{e}_z \times \left( \mathbf{H}_2(\mathbf{k}) - \mathbf{H}_1(\mathbf{k}) \right) = i\xi(\mathbf{q}_\perp) (\mathbf{e} - 1)(1 + \mathbb{R}_t) \hat{p}_0 / Z_0 = \tilde{J}_M / Z_0 \quad (H2.6) \]

Where

\[ \xi(\mathbf{q}_\perp) = \xi(\mathbf{k} - \mathbf{k}_\perp) = \frac{1}{(2\pi)^2} \iint d^2 r e^{-i(\mathbf{k} - \mathbf{k}_\perp) \mathbf{r'}} \xi(\mathbf{r'}) \quad (H2.7) \]

and \( \mathbf{q}_\perp = \mathbf{k} - \mathbf{k}_\perp \) is the projection on \( z=0 \) of the scattering vector \( \mathbf{q} \). It is seen that surface roughness in first order perturbation theory leads to the appearance of effective electric and magnetic currents \( \tilde{J}_E, \tilde{J}_M \) on the mean plane \( z=0 \), which generate the scattering fields in the upper and lower half-spaces.
Appendix H3 – Rayleigh hypothesis

The Rayleigh hypothesis makes use of a partial wave representation of the scattered fields in the upper (1) and lower (2) half-spaces evaluated at the boundary across media with different dielectric properties

\[
\begin{align*}
\tilde{E}_1(\vec{r}, z) = & \int\int \tilde{E}_1(\vec{k}) e^{ik_y} d^2k \\
\tilde{E}_2(\vec{r}, z) = & \int\int \tilde{E}_2(\vec{k}) e^{ik_y} d^2k
\end{align*}
\]

(H3.1)

where the Fourier amplitudes, \(E_1(k)\) and \(E_2(k)\), are determined by the requirements of the electromagnetic boundary conditions. Because planes waves are natural solutions to the homogeneous wave equation, the scattered fields in the upper and lower half-spaces are constructed using the continuations:

\[
\begin{align*}
\tilde{E}_1(\vec{r}, z) = & \int\int \tilde{E}_1(\vec{k}) e^{ik_y + \kappa z} d^2k & z \geq 0 & \text{with } \kappa_z = \sqrt{k^2 - \kappa^2} \\
\tilde{E}_2(\vec{r}, z) = & \int\int \tilde{E}_2(\vec{k}) e^{ik_y - \kappa z} d^2k & z < 0 & \text{and } \kappa_z' = \sqrt{ek^2 - \kappa^2}
\end{align*}
\]

(H3.2)

The validity of the “analytical continuation” is based on the fact that (H3.2) satisfies the corresponding wave equations and gives the correct value for the fields at the boundary.
Appendix I – Tilted plane perturbation theory

The tilted-plane perturbation theory for the two-scale model

The orientation of the local (facet) normal relative to the surface (mean plane) frame is given by the angles $\psi$ and $\delta$, such that $z'$ deviates from $z$ by $\psi$ in the plane of incidence (formed by unit vectors $n_{\text{inc}}$ and $z$) and by $\delta$ in a plane perpendicular to the latter. The relation between the local incidence angle $\beta$ and the global incidence angle $\theta$ is determined by (use the law of cosines in spherical right triangle above):

$$\cos \beta = \cos(\theta - \psi) \cos \delta$$  \hspace{1cm} (I.1)

So that the local angle of incidence remains a function of the facet tilt angles. The components of the facet normal in the global frame, $\hat{z}'$, are obtained by performing two ordered active rotations of the local frame vertical:

$$\hat{z}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} \cos \delta & 0 & \sin \delta \\ 0 & 1 & 0 \\ -\sin \delta & 0 & \cos \delta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \sin \delta \\ -\sin \psi \cos \delta \\ \cos \psi \cos \delta \end{bmatrix}$$ \hspace{1cm} (I.2)
By absorbing the $\psi$ tilt angle into the global incidence angle $\theta$, we can study the projections onto the local facet frame of polarization basis vectors expressed in the global frame by making $\psi = 0$ (here and below, $\theta \rightarrow \theta - \psi$):

\[
\begin{bmatrix}
\hat{h}' \\
\hat{v}'
\end{bmatrix} = U^{-1} \begin{bmatrix}
\hat{h} \\
\hat{v}
\end{bmatrix} = \begin{bmatrix}
\hat{h} \cdot \hat{h}' & \hat{h} \cdot \hat{v}' \\
\hat{v} \cdot \hat{h}' & \hat{v} \cdot \hat{v}'
\end{bmatrix} = \begin{bmatrix}
\sin \delta \sin(\theta - \psi) & \sin \delta \\
-\sin \delta & \cos \delta \sin(\theta - \psi)
\end{bmatrix} \begin{bmatrix}
\hat{h}' \\
\hat{v}'
\end{bmatrix}
\]

With (see Appendix F):

\[
\hat{h}' = \hat{x}' = \frac{\vec{n}_{\text{inc}} \times \hat{z}'}{\|\vec{n}_{\text{inc}} \times \hat{z}'\|} = \frac{1}{\sin \beta} \begin{bmatrix}
\cos \delta \sin(\theta - \psi) \\
-\sin \delta \cos \theta \\
-\sin \delta \sin \theta
\end{bmatrix}
\]

\[
\hat{v}' = \hat{x}' \times \vec{n}_{\text{inc}} = \frac{1}{\sin \beta} \begin{bmatrix}
\sin \delta \\
\cos \delta \sin(\theta - \psi) \cos \theta \\
\cos \delta \sin(\theta - \psi) \sin \theta
\end{bmatrix}
\]

Whereby

\[
U = \frac{1}{\sin \beta} \begin{bmatrix}
\cos \delta \sin(\theta - \psi) & -\sin \delta \\
\sin \delta & \cos \delta \sin(\theta - \psi)
\end{bmatrix}
\]

The transformation of basis of polarization is performed on the scattering amplitude matrix via the transformation matrix $U$ as (see Appendix J):

\[
\begin{bmatrix}
\tilde{F}_{hh} & \tilde{F}_{hv} \\
\tilde{F}_{vh} & \tilde{F}_{vv}
\end{bmatrix}_{\text{global frame}} = U \begin{bmatrix}
F_{hh} & F_{hv} \\
F_{vh} & F_{vv}
\end{bmatrix}_{\text{facet frame}} U^{-1} =
\]

\[
= \frac{1}{\sin^2 \beta} \begin{bmatrix}
F_{hh} (\sin(\theta - \psi) \cos \delta)^2 + F_{hv} (\sin \delta)^2 & F_{hv} (\sin(\theta - \psi) \cos \delta)^2 - F_{vh} (\sin \delta)^2 \\
-(F_{hv} + F_{vh}) \sin(\theta - \psi) \cos \delta \sin \delta & +(F_{hh} - F_{hv}) \sin(\theta - \psi) \cos \delta \sin \delta
\end{bmatrix}
\]

Which simplifies in first order backscatter geometry to [Valenzuela, 1977] (Note that in this case, incident and scattered angles in the local frame are identical to $\beta$ with $\phi_o = -\pi/2$, hence the local cross-polarized (SPM) scattering amplitudes vanish):
\[
\begin{bmatrix}
\tilde{F}_{hh} & \tilde{F}_{hv} \\
\tilde{F}_{vh} & \tilde{F}_{vv}
\end{bmatrix}_{\text{global frame}} = \frac{1}{\sin^2 \beta} \left[ F_{hh} (\sin(\theta - \psi) \cos \delta)^2 + F_{vv} (\sin \delta)^2 - (F_{hh} - F_{vv}) \sin(\theta - \psi) \cos \delta \sin \delta \right]
\]

For this simple (backscatter) case, the SPM cross-section in the global frame is dependent on the local facet tilt angles \(\psi, \delta\) through the local incidence angle \(\beta(\psi, \delta)\) with \(q_\perp(\psi, \delta) = 2k \sin \beta\):

\[
\sigma^0_{pq}(\beta, 0) = \frac{4\pi}{A} \left\{ |F_{pq}(\beta, 0)|^2 \right\} = \pi k^4 \left| e^{-1} \right|^2 \left| f_{qp}(\beta, 0) \right|^2 S(2k \sin \beta) \quad (I.7)
\]

To remove the statistical dependence on facet tilt angles, an ensemble average operation is performed over them using the large-scale roughness PDF of slopes in the global frame as:

\[
\left\langle \sigma^0_{pq} \right\rangle_{\text{tilt}} = \int d(\tan \psi) \int d(\tan \delta) \sigma^0_{pq}(\beta, 0) \, PDF_{\psi, \delta}(\tan \psi, \tan \delta) \quad (I.8)
\]

For the slightly more complicated general bistatic case, the tilted perturbation method demands that we express the local scattered angles \(\beta_o, \phi_o\) in terms of the local facet tilt angles \(\psi, \delta\) too. We do so by writing:

\[
\cos \beta_o = \hat{n}_{\text{scat}} \cdot \hat{z}' = \sin \delta \sin \theta_s \cos \phi_s - \sin \psi \cos \delta \sin \theta_s \sin \phi_s + \cos \theta_s \cos \psi \cos \delta
\]

\[
\cos \phi_o = \hat{x}' \cdot (\hat{n}_{\text{scat}} - (\hat{n}_{\text{scat}} \cdot \hat{z}') \hat{z}')
\]

\[
\sin \phi_o = \left| \hat{z}' \times (\hat{n}_{\text{scat}} - (\hat{n}_{\text{scat}} \cdot \hat{z}') \hat{z}') \right| \quad (I.9)
\]

And so in general we will have \(\beta(\psi, \delta), \beta_o(\psi, \delta), \phi_o(\psi, \delta)\) with a perturbation theory cross-section \(\sigma^0_{pq}(\beta, 0; \beta_o, \phi_o)\) given from (I.5) and averaged over the local facet tilt angles \(\psi, \delta\) as in the backscattering case.
Appendix J – Transformation between polarization bases

On the transformation between different electric field polarization bases

For the description of a general state of polarization, any set of two orthogonal unit vectors perpendicular to the direction of wave propagation is acceptable. If we assume propagation along the z-axis, then

\[ \hat{x} = \hat{h} \quad \hat{y} = \hat{v} \]  

(J.1)

form a basis that resolves the electric field into linear horizontal and vertically polarized components (i.e. a Jones vector). Another basis is formed by the complex orthogonal unit vectors

\[ \hat{c}_R = (\hat{h} - i\hat{v})/\sqrt{2} \quad \text{such that} \quad \hat{h} = (\hat{c}_R + \hat{c}_L)/\sqrt{2} \]
\[ \hat{c}_L = (\hat{h} + i\hat{v})/\sqrt{2} \quad \hat{v} = i(\hat{c}_R - \hat{c}_L)/\sqrt{2} \]  

(J.2)

which resolve the electric field into circular right-handed and left-handed polarized components. The transformation matrix between the linear and circular bases:

\[
\begin{bmatrix}
\hat{c}_R \\
\hat{c}_L 
\end{bmatrix} = U
\begin{bmatrix}
\hat{h} \\
\hat{v}
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} \hat{h} \\
\hat{v} \end{bmatrix}
\]  

(J.3)

Therefore, the transformation of electric field components is carried out as:

\[
\begin{bmatrix}
E_R \\
E_L
\end{bmatrix} = U^\dagger
\begin{bmatrix}
E_h \\
E_v
\end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \begin{bmatrix} E_h \\
E_v \end{bmatrix}
\]  

(J.4)

In the scattering matrix representation (\( F_{pp0} \) [Fung, 1994]):

\[
\begin{bmatrix}
E_h \\
E_v
\end{bmatrix}_{\text{scat}} = \frac{e^{i\phi_R}}{R} \begin{bmatrix}
F_{hh} & F_{hv} \\
F_{vh} & F_{vv}
\end{bmatrix} \begin{bmatrix}
E_h \\
E_v_{\text{inc}}
\end{bmatrix}
\]  

(J.5)

To effect a change of basis:

\[
\begin{bmatrix}
E_h \\
E_v
\end{bmatrix} = U
\begin{bmatrix}
E_R \\
E_L
\end{bmatrix}_{\text{scat}}
\]  

(J.6)
Then the scattering cross-section in a circular polarization basis \((p,q = L,R)\), in terms of the transformed scattering matrix: \((J.7)\)

\[
\sigma_{pq}^0(\theta, \phi) = \frac{4\pi}{A} \left| F_{pq}(\theta, \phi) \right|^2 = \frac{\pi}{A} \left[ \left| F_{hh} + F_{vv} \right|^2 + \left| F_{hv} - F_{vh} \right|^2 \left| F_{hh} - F_{vv} \right|^2 + \left| F_{hv} + F_{vh} \right|^2 \left| F_{hh} + F_{vv} \right|^2 + \left| F_{hv} - F_{vh} \right|^2 \right]
\]

Where in the Kirchhoff approximation:

\[
F_{qp} = -i k C_{pq} \frac{q}{q_z} e^{i q_z z} dS \quad (J.8)
\]

Which under scalar assumption \((mss<<1, \text{cross-polarized contributions vanish, see 5.1.10 in Chapter II})\) simplifies to: \((J.9)\)

\[
\sigma_{pq}^0(\theta, \phi) = \frac{4\pi}{A} \left( F_{pq}(\theta, \phi) \right)^2 = \frac{\pi}{A} \left( \frac{kq}{2q_z^2} \right)^2 \left[ C_{hh} + C_{vv} \right]^2 \left[ C_{hh} - C_{vv} \right]^2 PDF_{q_z} \frac{q_z}{q_z} (q_z, q_z)
\]

Whereas in the small perturbation method, the scattering amplitude is written as (see 5.2.5 in Chapter II):

\[
F_{qp} = -i \pi k^2 (e-1) f_{qp} \frac{\hat{q}}{q_z} (\hat{q}_z) \quad (J.10)
\]

And hence, the SPM cross-section in a circular polarization basis \((p,q = L,R)\): \((J.11)\)

\[
\sigma_{pq}^0(\theta, \phi) = \frac{\pi}{4} k^4 |e-1|^2 \left[ \left| f_{hh} + f_{vv} \right|^2 + \left| f_{hv} - f_{vh} \right|^2 \left| f_{hh} - f_{vv} \right|^2 + \left| f_{hv} + f_{vh} \right|^2 \left| f_{hh} + f_{vv} \right|^2 + \left| f_{hv} - f_{vh} \right|^2 \right] S(\hat{q}_z)
\]

Which also applies to the SSA cross-section, since it makes use of the same geometric pre-factors as the SPM cross-section.
Appendix K – Differentiability of a random field

The necessary and sufficient condition for the existence of the mean square derivative of a random field is that the first derivative of the autocorrelation be zero at zero lag. More explicitly, suppose a random function $X(t)$. The derivative of this random function:

$$
\dot{X}(t) = \frac{X(t + \Delta t) - X(t)}{\Delta t}
$$

such that $\sigma^2_x = \langle \dot{X}(t)\dot{X}(t) \rangle = \frac{2\sigma^2_x}{(\Delta t)^2} [1 - \rho(\Delta t)] \quad \text{(K.1)}$

The mean square derivative will exist only if the autocorrelation function has the following limit as $\Delta t \rightarrow 0$:

$$
\rho(\Delta t) = 1 - \frac{1}{2} \frac{\sigma^2_x}{\sigma^2_x (\Delta t)^2}
$$

Implying that the first derivative of the autocorrelation function must be zero at the origin for the mean square slope to be finite. A number of well known models, such as white noise or an exponentially correlated surface, do not satisfy this condition. A minimal amount of local averaging though, will render the process $X(t)$ mean square differentiable [Vandemarcke, 1984]. Define an averaged (i.e. upper-band limited) process $X_T(t)$:

$$
X_T(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} X(u) du \quad \text{(K.3)}
$$

Such that $\sigma^2_{x_T} = \gamma(T)\sigma^2_x$ where $\gamma(T) = \frac{1}{T^2} \int_0^T \int_0^T \rho(t_1 - t_2) dt_1 dt_2 \quad \text{(K.4)}$

In the following, we prove that:

i) The derivative of the autocorrelation of the averaged process $X_T(t)$ is zero at zero lag

From the algebraic identity (see Figure K.1 below),

$$
2I_TI_T = I_{I_0}^2 + I_{I_1}^2 + I_{I_2}^2 = 2 \langle X_T(t)X_T(t + \tau) \rangle T^2 \quad \text{where} \quad I_T = \int_T X(u) du
$$

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It follows that the autocorrelation function of the averaged process:

\[ \rho_T(\tau) = \frac{\{X_T(t)X_T(t+\tau)\}}{\sigma_T^2} = \]

\[ = \frac{1}{2T^2\gamma(T)}[(T-\tau)^2\gamma(T-\tau) - 2\tau^2\gamma(\tau) + (T+\tau)^2\gamma(T+\tau)] \]

(K.6)

With

\[ \lim_{T\to0} \rho_T(\tau) = \frac{1}{2} \frac{\partial^2}{\partial \tau^2} [\tau^2\gamma(\tau)] = \rho(\tau) \]  

(K.7)

And its derivative from (K.6),

\[ \frac{\partial}{\partial \tau} \rho_T(\tau) \bigg|_{\tau=0} = 0 \quad \text{for } T>0 \text{ and arbitrary } \gamma(T) \]  

(K.8)

\[ \frac{\partial^2}{\partial \tau^2} \rho_T(\tau) = \frac{\partial}{\partial \tau} \rho_T(\tau) \bigg|_{\tau=0} = 0 \quad \text{for } T>0 \text{ and arbitrary } \gamma(T) \]  

(K.8)

\[ \begin{aligned}
\text{Figure K.1 – The correlation of a locally averaged random function}
\end{aligned} \]

\[ \text{ii) The mean square slope of the averaged process } X_t(t) \text{ is given by} \]

\[ \sigma_{x_T}^2 = 2\sigma_X^2 / T^2 [1 - \rho(T)] \]

The mean square slope of the averaged process, from (K.6) and (K.7),

\[ \sigma_{x_T}^2 = -\sigma_{x_T}^2 \frac{\partial^2}{\partial \tau^2} \rho_T(\tau) = -\frac{\sigma_{x_T}^2}{T^2\gamma(T)} (\rho(T-\tau) - 2\rho(\tau) + \rho(T+\tau)) = \frac{2\sigma_X^2}{T^2} [1 - \rho(T)] \]  

(K.9)
Appendix L – NIC Sea ice charts

NIC ice analyses for the third week of March in 2003
Where the following codes are used to denote stages of sea ice development in NIC charts:

Table L.1 – Sea ice ‘egg codes’

<table>
<thead>
<tr>
<th>Stage of Development</th>
<th>Code Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Ice-Frazil, Grease, Slush, Shuga (0-10 cm)</td>
<td>1</td>
</tr>
<tr>
<td>Nilas, Ice Rind (0 - 10 cm)</td>
<td>2</td>
</tr>
<tr>
<td>Young (10 - 30 cm)</td>
<td>3</td>
</tr>
<tr>
<td>Gray (10 - 15 cm)</td>
<td>4</td>
</tr>
<tr>
<td>Gray - White (15 - 30 cm)</td>
<td>5</td>
</tr>
<tr>
<td>First Year (30 - 120 cm)</td>
<td>6</td>
</tr>
<tr>
<td>First Year Thin (30 - 70 cm)</td>
<td>7</td>
</tr>
<tr>
<td>First Year Thin- First Stage (30 - 70 cm)</td>
<td>8</td>
</tr>
<tr>
<td>First Year Thin- Second Stage (30 - 70 cm)</td>
<td>9</td>
</tr>
<tr>
<td>Med First Year (70 - 120 cm)</td>
<td>1*</td>
</tr>
<tr>
<td>Thick First Year (&gt;120 cm)</td>
<td>4*</td>
</tr>
<tr>
<td>Old-Survived at least one seasons melt (&gt;2 m)</td>
<td>7*</td>
</tr>
<tr>
<td>Second Year (&gt;2 m)</td>
<td>8*</td>
</tr>
<tr>
<td>Multi-Year (&gt;2 m)</td>
<td>9*</td>
</tr>
<tr>
<td>Predominant</td>
<td></td>
</tr>
<tr>
<td>Belt and strips</td>
<td>~</td>
</tr>
<tr>
<td>Landfast</td>
<td>8</td>
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