1. (35%) Given that the observations are related to the state by

\[ y_i = (t_i - 1)x_1 + (t_i^2 + 1)x_2 + \epsilon_i \quad i = 1, 2 \]

Observations \( y_i \) are taken at \( t_1=0, t_2=1 \), and \( y_1 \) is as accurate as \( y_2 \).

The values of the observations are: \( Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \). The state vector is \( X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \).

Assume \textit{a priori} information is given: \( X = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \), \( W = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \).

a. Write the observation-state equation in the form \( Y = HX + \epsilon \).

b. Compute the least squares estimate of \( X \) including the \textit{a priori} information.

c. Compute the best estimate of \( \epsilon \).

2. (35%) Given the system

\[ \begin{align*}
\dot{x}_1 &= \alpha x_1 + \beta x_2 \\
\dot{x}_2 &= \alpha x_2 
\end{align*} \]

a. Write the equations in state space form, \( \dot{X} = AX \), where \( X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \).

b. Determine the state transition matrix for this system.

c. Assuming \( t_0 = 0 \), write the expression for \( X \) at \( t = 1 \) in terms of the initial conditions \( x_1(t_0) \) and \( x_2(t_0) \).

3. (30%) Answer the following questions true or false.

a. In problem 1 the observation state relationship is nonlinear ______

b. If the state vector is \( n \times 1 \) and the observation vector in \( m \times 1 \) the H matrix will be \( m \times n \) ______

c. If there are fewer observations than unknowns but we are given \textit{apriori} state information with a full rank weighting matrix it is possible to obtain a least squares estimate for the state ______

d. Range observations of a satellite from two different ground stations at the same instant in time generally will not be independent ______

e. The differential equation \( \ddot{x} + 3\dot{x} + 2x^2 = 0 \) is linear ______